Ch13.4 Randomized Block Design

- Experimental units are the objects of interest in the experiment.
- A completely randomized design is an experimental design in which the treatments are randomly assigned to the experimental units.
- If the experimental units are heterogeneous, blocking can be used to form homogeneous groups, resulting in a randomized block design.

課本範例說明 (p.621~)

- **Example：Air Traffic Controller Stress Test**
  - A study measuring the fatigue and stress of air traffic controllers resulted in proposals for modification and redesign of the controller's workstation. After consideration of several designs for the workstation, three specific alternatives are selected as having the best potential for reducing controller stress. The key question is: To what extent do the three alternatives differ in terms of their effect on controller stress？
  - Controllers are believed to differ substantially in their ability to handle stressful situations.
  - When considering the within-group source of variation (MSE), we must realize that this variation includes both random error and error due to individual controller differences.
  - Each controller in the sample is tested with each of the three workstation alternatives → **blocks**

		Treatments		
		System A	System B	System C
Blocks	Controller 1	15	15	18
	Controller 2	14	14	14
	Controller 3	10	11	15
	Controller 4	13	12	17
	Controller 5	16	13	16
	Controller 6	13	13	13

pp.623-624

x_{ij} = value of the observation corresponding to treatment j in block i
$\bar{x}_{\cdot j}$ = sample mean of the jth treatment
$\bar{x}_{i \cdot}$ = sample mean for the ith block
$\bar{\bar{x}}$ = overall sample mean

Step 1. Compute the total sum of squares (SST).

$$SST = \sum_{i=1}^{b}\sum_{j=1}^{k}(x_{ij} - \bar{\bar{x}})^2 \qquad (13.22)$$

Step 2. Compute the sum of squares due to treatments (SSTR).

$$SSTR = b\sum_{j=1}^{k}(\bar{x}_{\cdot j} - \bar{\bar{x}})^2 \qquad (13.23)$$

Step 3. Compute the sum of squares due to blocks (SSBL).

$$SSBL = k\sum_{i=1}^{b}(\bar{x}_{i \cdot} - \bar{\bar{x}})^2 \qquad (13.24)$$

Step 4. Compute the sum of squares due to error (SSE).

$$SSE = SST - SSTR - SSBL \qquad (13.25)$$

		Treatments			Row or Block Totals	Block Means
		System A	System B	System C		
Blocks	Controller 1	15	15	18	48	$\bar{x}_{1\cdot} = 48/3 = 16.0$
	Controller 2	14	14	14	42	$\bar{x}_{2\cdot} = 42/3 = 14.0$
	Controller 3	10	11	15	36	$\bar{x}_{3\cdot} = 36/3 = 12.0$
	Controller 4	13	12	17	42	$\bar{x}_{4\cdot} = 42/3 = 14.0$
	Controller 5	16	13	16	45	$\bar{x}_{5\cdot} = 45/3 = 15.0$
	Controller 6	13	13	13	39	$\bar{x}_{6\cdot} = 39/3 = 13.0$
Column or Treatment Totals		81	78	93	252	$\bar{\bar{x}} = \frac{252}{18} = 14.0$
Treatment Means		$\bar{x}_{\cdot 1} = \frac{81}{6} = 13.5$	$\bar{x}_{\cdot 2} = \frac{78}{6} = 13.0$	$\bar{x}_{\cdot 3} = \frac{93}{6} = 15.5$		

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	SSTR	$k - 1$	$\text{MSTR} = \frac{\text{SSTR}}{k - 1}$	$\frac{\text{MSTR}}{\text{MSE}}$	
Blocks	SSBL	$b - 1$	$\text{MSBL} = \frac{\text{SSBL}}{b - 1}$		
Error	SSE	$(k - 1)(b - 1)$	$\text{MSE} = \frac{\text{SSE}}{(k - 1)(b - 1)}$		
Total	SST	$n_T - 1$			

For the air traffic controller data in Table 13.6, these steps lead to the following sums of squares.

Step 1. $\text{SST} = (15 - 14)^2 + (15 - 14)^2 + (18 - 14)^2 + \cdots + (13 - 14)^2 = 70$

Step 2. $\text{SSTR} = 6[(13.5 - 14)^2 + (13.0 - 14)^2 + (15.5 - 14)^2] = 21$

Step 3. $\text{SSBL} = 3[(16 - 14)^2 + (14 - 14)^2 + (12 - 14)^2 + (14 - 14)^2 + (15 - 14)^2 + (13 - 14)^2] = 30$

Step 4. $\text{SSE} = 70 - 21 - 30 = 19$

TABLE 13.8 ANOVA TABLE FOR THE AIR TRAFFIC CONTROLLER STRESS TEST

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	21	2	10.5	$10.5/1.9 = 5.53$	.024
Blocks	30	5	6.0		
Error	19	10	1.9		
Total	70	17			

These sums of squares divided by their degrees of freedom provide the corresponding mean square values shown in Table 13.8.

Let us use a level of significance $\alpha = .05$ to conduct the hypothesis test. The value of the test statistic is

$$F = \frac{\text{MSTR}}{\text{MSE}} = \frac{10.5}{1.9} = 5.53$$

The p-value ≤ $\alpha = .05$ (where $F = 5.53$). (Excel provides a p-value of .024). Therefore, we reject H_0 : $\mu_1 = \mu_2 = \mu_3$ and conclude that the population mean stress levels differ for the three workstation alternatives.

26. **SAT Performance.** The Scholastic Aptitude Test (SAT) contains three areas: critical reading, mathematics, and writing. Each area is scored on an 800-point scale. A sample of SAT scores for six students follows.

Student	Critical Reading	Mathematics	Writing
1	526	534	530
2	594	590	586
3	465	464	445
4	561	566	553
5	436	478	430
6	430	458	420

a. Using a .05 level of significance, do students perform differently on the three areas of the SAT?
b. Which area of the test seems to give the students the most trouble? Explain.

a. Treatment Means:

$\bar{x}_{\cdot 1} = 502$ $\bar{x}_{\cdot 2} = 515$ $\bar{x}_{\cdot 3} = 494$

Block Means:

$\bar{x}_{1\cdot} = 530$ $\bar{x}_{2\cdot} = 590$ $\bar{x}_{3\cdot} = 458$ $\bar{x}_{4\cdot} = 560$ $\bar{x}_{5\cdot} = 448$ $\bar{x}_{6\cdot} = 436$

Overall Mean:

$\bar{\bar{x}} = 9066/18 = 503.67$

Step 1

$$\text{SST} = \sum_i \sum_j \left(x_{ij} - \bar{\bar{x}}\right)^2 = (526 - 503.67)^2 + (534 - 503.67)^2 + \cdot\cdot\cdot + (420 - 503.67)^2 = 65{,}798$$

Step 2

$$\text{SSTR} = b\sum_j \left(\bar{x}_{\cdot j} - \bar{\bar{x}}\right)^2 = 6[\,(502 - 503.67)^2 + (515 - 503.67)^2 + (494 - 503.67)^2\,] = 1348$$

Step 3

$$\text{SSBL} = k\sum_i \left(\bar{x}_{i\cdot} - \bar{\bar{x}}\right)^2 = 3\,[\,(530 - 503.67)^2 + (590 - 503.67)^2 + \cdot\cdot\cdot + (436 - 503.67)^2\,] = 63{,}250$$

Step 4

SSE = SST – SSTR – SSBL = 65,798 – 1348 – 63,250= 1200

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	1348	2	674	5.62	.0231
Blocks	63,250	5	12,650		
Error	1200	10	120		
Total	65,798	17			

Using F table (2 degrees of freedom numerator and 10 denominator), p-value is between .01 and .025.

(b)

The mean test scores for the three sections are 502 for critical reading, 515 for mathematics, and 494 for writing. Because the writing section has the lowest average score, this section appears to give the students the most trouble.

Homework 3 (另有題目檔案提供)