110-2 (2022.5.10)

13.3 Multiple Comparison Procedures (p.615)

- Suppose that analysis of variance has provided statistical evidence to reject the null hypothesis of equal population means.

- Fisher's least significant difference (LSD) procedure can be used to determine where the differences occur.

Hypotheses:

$$H_0\colon \mu_i = \mu_j$$
$$H_a\colon \mu_i \neq \mu_j$$

Test Statistic:

$$t = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$$

p-value approach: Reject H_0 if the *p*-value ≤ 0.05

Critical value approach: Reject H_0 if $t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$

Where the value of $t_{\alpha/2}$ is based on a t distribution with $n_T - k$ degrees of freedom.

Go back to 13.1 example (p.600)

	Method		
	A	B	C
	58	58	48
	64	69	57
	55	71	59
	66	64	47
	67	68	49
Sample mean	62	66	52
Sample variance	27.5	26.5	31.0
Sample standard deviation	5.244	5.148	5.568

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	520	2	260.00	9.18	.004
Error	340	12	28.33		
Total	860	14			

Now, let's compare method A and method B

$$t = \frac{62 - 66}{\sqrt{28.33\left(\frac{1}{5} + \frac{1}{5}\right)}} = -1.19$$

Because we have a two-tailed test, the p-value is two times the area under the curve for the t distribution to the left of t = −1.19.

Area in Upper Tail	.20	.10	.05	.025	.01	.005
χ^2 Value (293 *df*)	.873	1.356	1.782	2.179	2.681	3.055

$t = 1.19$

Doubling these amounts, we see that the p-value is greater than α = .05, we cannot reject the null hypothesis. So, is there any difference of population means between method A and method B?

110-2 (2022.5.10)

(老師補充) 其實用 critical value 檢驗的概念來理解 LSD 比較容易

$|t| \geq t_{\alpha/2}$ 拒絕 H0, Ha 成立，兩個母體平均數有差異

p.616 Fisher's LSD procedure based on the test statistic $\bar{x}_i - \bar{x}_j$

- **Hypotheses** $H_0: \mu_i = \mu_j$, $H_a: \mu_i \neq \mu_j$
- **Test Statistic** $\bar{x}_i - \bar{x}_j$
- **Rejection Rule** Reject H_0 if $|\bar{x}_i - \bar{x}_j| > \text{LSD}$

where $\text{LSD} = t_{\alpha/2}\sqrt{\text{MSE}(1/n_i + 1/n_j)}$

$$\text{LSD} = 2.179\sqrt{28.33\left(\frac{1}{5}+\frac{1}{5}\right)} = 7.34$$

Method A vs. Method B

Method A vs. Method C

Method B vs. Method C

Exercise 14 (p.619)

14. The following data are from a completely randomized design. In the following calculations, use $\alpha = .05$.

	Treatment 1	Treatment 2	Treatment 3
	63	82	69
	47	72	54
	54	88	61
	40	66	48
$\bar{x}_j$	51	77	58
s_j^2	96.67	97.34	81.99

a. Use analysis of variance to test for a significant difference among the means of the three treatments.

b. Use Fisher's LSD procedure to determine which means are different.