期望值

$$\mathrm{E}(\overline{x_1}-\overline{x_2}) = E(\overline{x_1}) - E(\overline{x_2}) = \mu_1 - \mu_2$$

變異數

$$\begin{aligned}
\mathrm{Var}(\overline{x_1}-\overline{x_2}) &= \mathrm{E}\{[(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)]^2\} = E\{[(\overline{x_1}-\mu_1)+(\overline{x_2}-\mu_2)]^2\} \\
&= E[(\overline{x_1}-\mu_1)^2+(\overline{x_2}-\mu_2)^2+2(\overline{x_1}-\mu_1)(\overline{x_2}-\mu_2)] \\
&= E[(\overline{x_1}-\mu_1)^2+(\overline{x_2}-\mu_2)^2+0] \\
&= E\left[\left(\frac{x_{11}+x_{12}+\cdots+x_{1n}}{n_1}-\mu_1\right)^2+\left(\frac{x_{21}+x_{22}+\cdots+x_{2n}}{n_2}-\mu_2\right)^2\right] \\
&= \frac{1}{n_1^2}E[(x_{11}+x_{12}+\cdots+x_{1n})-n\mu_1)^2] \\
&+ \frac{1}{n_2^2}E[(x_{21}+x_{22}+\cdots+x_{2n})-n\mu_2)^2] \\
&= \frac{1}{n_1^2}\{E[(x_{11}-\mu_1)^2]+E[(x_{12}-\mu_1)^2]+\cdots+E[(x_{1n}-\mu_1)^2]\} \\
&+ \frac{1}{n_2^2}\{E[(x_{21}-\mu_1)^2]+E[(x_{22}-\mu_1)^2]+\cdots+E[(x_{2n}-\mu_1)^2]\} \\
&= \frac{1}{n_1^2}\cdot n_1\sigma_1^2+\frac{1}{n_2^2}\cdot n_2\sigma_2^2 = \frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}
\end{aligned}$$

因此，$\mathrm{Std}(\overline{x_1}-\overline{x_2}) = \sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$