Calculus(II) HW4 (3/19)

Sec. 4.5 # 10

Evaluate the indefinite integral.

$$\int \sin t\sqrt{1+\cos t}\ dt$$

[Solution]

Let $u = 1 + \cos t$. Then $du = -\sin t\,dt$ and $\sin t\,dt = -du$, so

$\int \sin t\,\sqrt{1+\cos t}\,dt = \int \sqrt{u}\,(-du) = -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}(1+\cos t)^{3/2} + C.$

Sec. 4.5 # 16

Evaluate the indefinite integral.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

[Solution]

Let $u = \sqrt{x}$. Then $du = \dfrac{1}{2\sqrt{x}}\,dx$ and $2\,du = \dfrac{1}{\sqrt{x}}\,dx$, so

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin u\,(2\,du) = -2\cos u + C = -2\cos\sqrt{x} + C.$$

Sec. 4.5 # 18

Evaluate the indefinite integral.

$$\int \sin x\,\sin(\cos x)\,dx$$

[Solution]

Let $u = \cos x$. Then $du = -\sin x\,dx$ and $-du = \sin x\,dx$, so

$\int \sin x\,\sin(\cos x)\,dx = \int \sin u\,(-du) = (-\cos u)(-1) + C = \cos(\cos x) + C.$

Sec. 4.5 # 23

Evaluate the indefinite integral.

$$\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz$$

[Solution]

Let $u = 1 + z^3$. Then $du = 3z^2\,dz$ and $z^2\,dz = \frac{1}{3}\,du$, so

$$\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz = \int u^{-1/3}\left(\tfrac{1}{3}\,du\right) = \tfrac{1}{3}\cdot\tfrac{3}{2}u^{2/3} + C = \tfrac{1}{2}(1+z^3)^{2/3} + C.$$

Sec. 4.5 # 38

Evaluate the definite integral.

$$\int_0^{\sqrt{\pi}} x \cos(x^2)\, dx$$

[Solution]

Let $u = x^2$, so $du = 2x\,dx$. When $x = 0$, $u = 0$; when $x = \sqrt{\pi}$, $u = \pi$. Thus,

$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \int_0^{\pi} \cos u \left(\frac{1}{2}\,du\right) = \frac{1}{2}\left[\sin u\right]_0^{\pi} = \frac{1}{2}(\sin\pi - \sin 0) = \frac{1}{2}(0-0) = 0.$

Sec. 4.5 # 43

Evaluate the definite integral.

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$$

[Solution]

Let $u = 1 + 2x$, so $du = 2\,dx$. When $x = 0$, $u = 1$; when $x = 13$, $u = 27$. Thus,

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^{27} u^{-2/3}\left(\frac{1}{2}\,du\right) = \left[\frac{1}{2}\cdot 3u^{1/3}\right]_1^{27} = \frac{3}{2}(3-1) = 3.$$

Sec. 4.5 # 49

Evaluate the definite integral.

$$\int_{1/2}^{1} \frac{\cos(x^{-2})}{x^3}\,dx$$

[Solution]

Let $u = x^{-2}$, so $du = -2x^{-3}\,dx$. When $x = \frac{1}{2}$, $u = 4$; when $x = 1$, $u = 1$. Thus,

$$\int_{1/2}^{1} \frac{\cos(x^{-2})}{x^3}\,dx = \int_4^1 \cos u\left(\frac{du}{-2}\right) = \frac{1}{2}\int_1^4 \cos u\,du = \frac{1}{2}\left[\sin u\right]_1^4 = \frac{1}{2}(\sin 4 - \sin 1).$$