Calculus(II) HW6 (04/09)

Sec.5.3 # 4

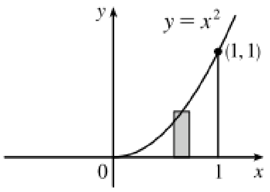
3–7 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curve about the y-axis.

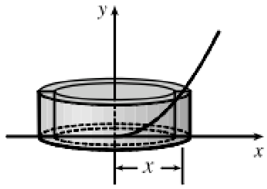
$y = x^2, \quad y = 0, \quad x = 1$

[Solution]

$$V = \int_0^1 2\pi x \cdot x^2 \, dx = 2\pi \int_0^1 x^3 \, dx$$

$$= 2\pi \left[\tfrac{1}{4}x^4\right]_0^1 = 2\pi \cdot \tfrac{1}{4} = \tfrac{\pi}{2}$$





Sec.5.3 # 6

3–7 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curve about the y-axis.

$y = 4x - x^2, \quad y = x$

[Solution]

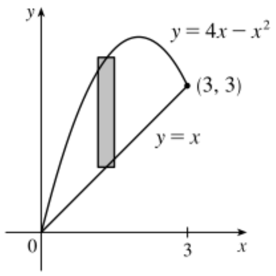
$4x - x^2 = x \quad \Leftrightarrow \quad 0 = x^2 - 3x \quad \Leftrightarrow \quad 0 = x(x-3) \quad \Leftrightarrow \quad x = 0$ or 3.

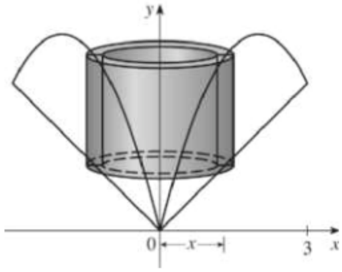
$$V = \int_0^3 2\pi x[(4x - x^2) - x] \, dx$$

$$= 2\pi \int_0^3 (-x^3 + 3x^2) \, dx$$

$$= 2\pi \left[-\tfrac{1}{4}x^4 + x^3\right]_0^3$$

$$= 2\pi\left(-\tfrac{81}{4} + 27\right) = 2\pi\left(\tfrac{27}{4}\right) = \tfrac{27}{2}\pi$$





Sec.5.3 # 11

9–14 Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.

$y = x^{3/2}, \quad y = 8, \quad x = 0$

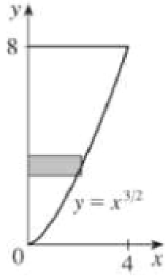
[Solution]

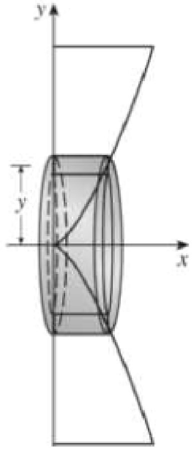
$y = x^{3/2} \quad \Rightarrow \quad x = y^{2/3}$. The shell has radius y, circumference $2\pi y$, and height $y^{2/3}$, so

$$V = \int_0^8 2\pi y(y^{2/3})\, dy = 2\pi \int_0^8 y^{5/3}\, dy$$

$$= 2\pi \left[\tfrac{3}{8} y^{8/3}\right]_0^8$$

$$= 2\pi \cdot \tfrac{3}{8} \cdot 256 = 192\pi$$





Sec.5.3 # 15

15–20 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

$y = x^3, \; y = 8, \; x = 0; \quad$ about $x = 3$

[Solution]

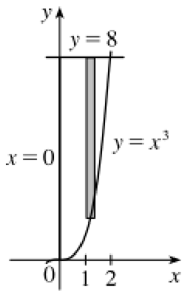
The shell has radius $3 - x$, circumference $2\pi(3 - x)$, and height $8 - x^3$.

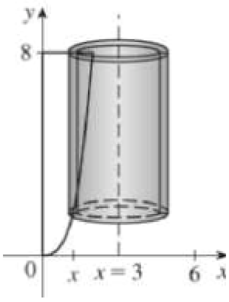
$$V = \int_0^2 2\pi(3 - x)(8 - x^3)\, dx$$

$$= 2\pi \int_0^2 (x^4 - 3x^3 - 8x + 24)\, dx$$

$$= 2\pi \left[\tfrac{1}{5}x^5 - \tfrac{3}{4}x^4 - 4x^2 + 24x\right]_0^2$$

$$= 2\pi\left(\tfrac{32}{5} - 12 - 16 + 48\right) = 2\pi\left(\tfrac{132}{5}\right) = \tfrac{264\pi}{5}$$





Sec.6.1 # 11

3–16 A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

$g(x) = 1 - \sin x$

[Solution]

$g(x) = 1 - \sin x. \quad g(0) = 1$ and $g(\pi) = 1$, so g is not one-to-one.

Sec.6.1 # 24

23–28 Find a formula for the inverse of the function.

$f(x) = \dfrac{4x - 1}{2x + 3}$

$3y + 1 = (4 - 2y)x \quad \Rightarrow \quad x = \dfrac{3y+1}{4-2y}$. Interchange x and y: $y = \dfrac{3x+1}{4-2x}$. So $f^{-1}(x) = \dfrac{3x+1}{4-2x}$.

Sec.6.1 # 25

23–28 Find a formula for the inverse of the function.

$f(x) = 1 + \sqrt{2 + 3x}$

[Solution]

$y = f(x) = 1 + \sqrt{2+3x} \quad (y \geq 1) \quad \Rightarrow \quad y - 1 = \sqrt{2+3x} \quad \Rightarrow \quad (y-1)^2 = 2 + 3x \quad \Rightarrow \quad (y-1)^2 - 2 = 3x \quad \Rightarrow$
$x = \frac{1}{3}(y-1)^2 - \frac{2}{3}$. Interchange x and y: $y = \frac{1}{3}(x-1)^2 - \frac{2}{3}$. So $f^{-1}(x) = \frac{1}{3}(x-1)^2 - \frac{2}{3}$. Note that the domain of f^{-1} is $x \geq 1$.

Sec.6.1 # 28

23–28 Find a formula for the inverse of the function.

$f(x) = 2x^2 - 8x, \ x \geqslant 2$

[Solution]

$y = f(x) = 2x^2 - 8x, x \geq 2 \quad \Rightarrow \quad 2x^2 - 8x - y = 0, x \geq 2 \quad \Rightarrow$

$x = \dfrac{8 + \sqrt{64 + 8y}}{4} \left[\begin{array}{c}\text{quadratic formula with} \\ a = 2, b = -8, \text{ and } c = -y\end{array}\right] = \dfrac{8 + 2\sqrt{16+2y}}{4} = 2 + \frac{1}{2}\sqrt{16+2y}$. Interchange x and y:

$y = 2 + \frac{1}{2}\sqrt{16+2x}$. So $f^{-1}(x) = 2 + \frac{1}{2}\sqrt{16+2x}$.

Alternate solution (by completing the square): $y = 2x^2 - 8x, x \geq 2 \quad \Rightarrow \quad x^2 - 4x = y/2, x \geq 2 \quad \Rightarrow$

$(x-2)^2 = x^2 - 4x + 4 = \dfrac{y}{2} + 4 = \dfrac{y+8}{2} = \dfrac{2y+16}{4}, x \geq 2 \quad \Rightarrow \quad x - 2 = +\sqrt{\dfrac{2y+16}{4}} \quad \Rightarrow \quad x = 2 + \frac{1}{2}\sqrt{2y+16}$.

Interchange x and y: $y = 2 + \frac{1}{2}\sqrt{2x+16}$. So $f^{-1}(x) = 2 + \frac{1}{2}\sqrt{2x+16}$.