

Calculus(II) Quiz11(06/11)

1.

Evaluate the integral.

$$\int_0^{\pi} \cos^4(2t) dt$$

[Solution]

$$\begin{aligned} \int_0^{\pi} \cos^4(2t) dt &= \int_0^{\pi} [\cos^2(2t)]^2 dt = \int_0^{\pi} \left[\frac{1}{2}(1 + \cos(2 \cdot 2t)) \right]^2 dt \quad [\text{half-angle identity}] \\ &= \frac{1}{4} \int_0^{\pi} [1 + 2 \cos 4t + \cos^2(4t)] dt = \frac{1}{4} \int_0^{\pi} [1 + 2 \cos 4t + \frac{1}{2}(1 + \cos 8t)] dt \\ &= \frac{1}{4} \int_0^{\pi} \left(\frac{3}{2} + 2 \cos 4t + \frac{1}{2} \cos 8t \right) dt = \frac{1}{4} \left[\frac{3}{2}t + \frac{1}{2} \sin 4t + \frac{1}{16} \sin 8t \right]_0^{\pi} = \frac{1}{4} \left[\left(\frac{3}{2}\pi + 0 + 0 \right) - 0 \right] = \frac{3}{8}\pi \end{aligned}$$

2.

Evaluate the integral.

$$\int \tan^2 \theta \sec^4 \theta d\theta$$

[Solution]

$$\begin{aligned} \int \tan^2 \theta \sec^4 \theta d\theta &= \int \tan^2 \theta \sec^2 \theta \sec^2 \theta d\theta = \int \tan^2 \theta (\tan^2 \theta + 1) \sec^2 \theta d\theta \\ &= \int u^2(u^2 + 1) du \quad [u = \tan \theta, du = \sec^2 \theta d\theta] \\ &= \int (u^4 + u^2) du = \frac{1}{5}u^5 + \frac{1}{3}u^3 + C = \frac{1}{5} \tan^5 \theta + \frac{1}{3} \tan^3 \theta + C \end{aligned}$$