Calculus(II) HW3 (3/12)

Sec. 4.3 # 16

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_0^{\tan x} \sqrt{t + \sqrt{t}}\, dt$$

[Solution]

Let $u = \tan x$. Then $\frac{du}{dx} = \sec^2 x$. Also, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, so

$$y' = \frac{d}{dx}\int_0^{\tan x} \sqrt{t+\sqrt{t}}\,dt = \frac{d}{du}\int_0^{u} \sqrt{t+\sqrt{t}}\,dt \cdot \frac{du}{dx} = \sqrt{u+\sqrt{u}}\,\frac{du}{dx} = \sqrt{\tan x + \sqrt{\tan x}}\,\sec^2 x.$$

Sec. 4.3 # 17

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_{\sqrt{x}}^{\pi/4} \theta \tan\theta\, d\theta$$

[Solution]

Let $u = \sqrt{x}$. Then $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$. Also, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, so

$$y' = \frac{d}{dx}\int_{\sqrt{x}}^{\pi/4} \theta\tan\theta\,d\theta = -\frac{d}{du}\int_{\pi/4}^{\sqrt{x}} \theta\tan\theta\,d\theta \cdot \frac{du}{dx} = -u\tan u\,\frac{du}{dx} = -\sqrt{x}\tan\sqrt{x}\cdot\frac{1}{2\sqrt{x}} = -\frac{1}{2}\tan\sqrt{x}$$

Sec. 4.3 # 21

Evaluate the integral.

$$\int_0^2 \left(\tfrac{4}{5}t^3 - \tfrac{3}{4}t^2 + \tfrac{2}{5}t\right) dt$$

[Solution]

$$\int_0^2 \left(\tfrac{4}{5}t^3 - \tfrac{3}{4}t^2 + \tfrac{2}{5}t\right) dt = \left[\tfrac{1}{5}t^4 - \tfrac{1}{4}t^3 + \tfrac{1}{5}t^2\right]_0^2 = \left(\tfrac{16}{5} - 2 + \tfrac{4}{5}\right) - 0 = 2$$

Sec. 4.3 # 25

Evaluate the integral.

$$\int_1^2 \frac{3}{t^4}\, dt$$

[Solution]

$$\int_1^2 \frac{3}{t^4}\,dt = 3\int_1^2 t^{-4}\,dt = 3\left[\frac{t^{-3}}{-3}\right]_1^2 = \frac{3}{-3}\left[\frac{1}{t^3}\right]_1^2 = -1\left(\frac{1}{8} - 1\right) = \frac{7}{8}$$

Sec. 4.3 # 28

Evaluate the integral.

$$\int_0^1 (3 + x\sqrt{x})\,dx$$

[Solution]

$$\int_0^1 (3 + x\sqrt{x})\,dx = \int_0^1 (3 + x^{3/2})\,dx = \left[3x + \tfrac{2}{5}x^{5/2}\right]_0^1 = \left[\left(3 + \tfrac{2}{5}\right) - 0\right] = \tfrac{17}{5}$$

Sec. 4.3 # 53

Find the derivative of the function.

$$g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\,du$$

$$\left[\textit{Hint}: \int_{2x}^{3x} f(u)\,du = \int_{2x}^{0} f(u)\,du + \int_0^{3x} f(u)\,du\right]$$

[Solution]

$$g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du = \int_{2x}^{0} \frac{u^2-1}{u^2+1}\,du + \int_0^{3x} \frac{u^2-1}{u^2+1}\,du = -\int_0^{2x} \frac{u^2-1}{u^2+1}\,du + \int_0^{3x} \frac{u^2-1}{u^2+1}\,du \quad \Rightarrow$$

$$g'(x) = -\frac{(2x)^2-1}{(2x)^2+1} \cdot \frac{d}{dx}(2x) + \frac{(3x)^2-1}{(3x)^2+1} \cdot \frac{d}{dx}(3x) = -2 \cdot \frac{4x^2-1}{4x^2+1} + 3 \cdot \frac{9x^2-1}{9x^2+1}$$

Sec. 4.3 # 56

Find the derivative of the function.

$$g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}}\,dt$$

[Solution]

$$g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt = \int_{\tan x}^{1} \frac{dt}{\sqrt{2+t^4}} + \int_1^{x^2} \frac{dt}{\sqrt{2+t^4}} = -\int_1^{\tan x} \frac{dt}{\sqrt{2+t^4}} + \int_1^{x^2} \frac{dt}{\sqrt{2+t^4}} \quad \Rightarrow$$

$$g'(x) = \frac{-1}{\sqrt{2+\tan^4 x}} \frac{d}{dx}(\tan x) + \frac{1}{\sqrt{2+x^8}} \frac{d}{dx}(x^2) = -\frac{\sec^2 x}{\sqrt{2+\tan^4 x}} + \frac{2x}{\sqrt{2+x^8}}$$

Sec. 4.4 # 9

Find the general indefinite integral.

$$\int v(v^2 + 2)^2\,dv$$

[Solution]

$$\int v(v^2+2)^2\,dv = \int v(v^4 + 4v^2 + 4)\,dv = \int (v^5 + 4v^3 + 4v)\,dv = \frac{v^6}{6} + 4\frac{v^4}{4} + 4\frac{v^2}{2} + C = \tfrac{1}{6}v^6 + v^4 + 2v^2 + C$$

Sec. 4.4 # 11

Find the general indefinite integral.

$$\int \frac{1 + \sqrt{x} + x}{\sqrt{x}}\, dx$$

[Solution]

$$\int \frac{1+\sqrt{x}+x}{\sqrt{x}}\, dx = \int \left(\frac{1}{\sqrt{x}} + 1 + \sqrt{x}\right) dx = \int (x^{-1/2} + 1 + x^{1/2})\, dx$$

$$= 2x^{1/2} + x + \tfrac{2}{3}x^{3/2} + C = 2\sqrt{x} + x + \tfrac{2}{3}x^{3/2} + C$$

Sec. 4.4 # 16

Find the general indefinite integral.

$$\int \frac{\sin 2x}{\sin x}\, dx$$

[Solution]

$$\int \frac{\sin 2x}{\sin x}\, dx = \int \frac{2\sin x\, \cos x}{\sin x}\, dx = \int 2\cos x\, dx = 2\sin x + C$$

Sec. 4.4 # 25

Evaluate the integral.

$$\int_0^{\pi} (4\sin\theta - 3\cos\theta)\, d\theta$$

[Solution]

$$\int_0^{\pi} (4\sin\theta - 3\cos\theta)\, d\theta = \big[-4\cos\theta - 3\sin\theta\big]_0^{\pi} = (4-0) - (-4-0) = 8$$

Sec. 4.4 # 28

Evaluate the integral.

$$\int_1^2 \left(2 - \frac{1}{p^2}\right)^2 dp$$

[Solution]

$$\int_1^2 \left(2 - \frac{1}{p^2}\right)^2 dp = \int_1^2 \left(4 - \frac{4}{p^2} + \frac{1}{p^4}\right) dp = \int_1^2 (4 - 4p^{-2} + p^{-4})\, dp = \left[4p + 4p^{-1} - \frac{1}{3}p^{-3}\right]_1^2$$

$$= \left(8 + 2 - \frac{1}{24}\right) - \left(4 + 4 - \frac{1}{3}\right) = 2 - \frac{1}{24} + \frac{1}{3} = \frac{48 - 1 + 8}{24} = \frac{55}{24}$$

Sec. 4.4 # 34

Evaluate the integral.

$$\int_0^{\pi/3} \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta}\, d\theta$$

[Solution]

$$\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\, \tan^2\theta}{\sec^2\theta}\, d\theta = \int_0^{\pi/3} \frac{\sin\theta\,(1+\tan^2\theta)}{\sec^2\theta}\, d\theta = \int_0^{\pi/3} \frac{\sin\theta\, \sec^2\theta}{\sec^2\theta}\, d\theta = \int_0^{\pi/3} \sin\theta\, d\theta$$

$$= \left[-\cos\theta\right]_0^{\pi/3} = -\tfrac{1}{2} - (-1) = \tfrac{1}{2}$$

Sec. 4.4 # 39

Evaluate the integral.

$$\int_2^5 |x-3|\, dx$$

[Solution]

$$|x-3| = \begin{cases} x-3 & \text{if } x-3 \ge 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases} = \begin{cases} x-3 & \text{if } x \ge 3 \\ 3-x & \text{if } x < 3 \end{cases}$$

Thus,

$$\int_2^5 |x-3|\; dx = \int_2^3 (3-x)\, dx + \int_3^5 (x-3)\, dx = \left[3x - \tfrac{1}{2}x^2\right]_2^3 + \left[\tfrac{1}{2}x^2 - 3x\right]_3^5$$

$$= \left(9 - \tfrac{9}{2}\right) - (6-2) + \left(\tfrac{25}{2} - 15\right) - \left(\tfrac{9}{2} - 9\right) = \tfrac{5}{2}$$