Calculus(II) HW12 (05/28)

Sec.6.8 # 30

8–68 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x\to 0} \frac{x - \sin x}{x - \tan x}$$

[Solution]

This limit has the form $\frac{0}{0}$.

$$\lim_{x\to 0} \frac{x-\sin x}{x-\tan x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{1-\cos x}{1-\sec^2 x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{-(-\sin x)}{-2\sec x\,(\sec x\ \tan x)} = -\frac{1}{2}\lim_{x\to 0} \frac{\sin x\left(\dfrac{\cos x}{\sin x}\right)}{\sec^2 x}$$

$$= -\tfrac{1}{2}\lim_{x\to 0} \cos^3 x = -\tfrac{1}{2}(1)^3 = -\tfrac{1}{2}$$

Another method is to write the limit as $\displaystyle\lim_{x\to 0} \frac{1-\dfrac{\sin x}{x}}{1-\dfrac{\tan x}{x}}$.

Sec.6.8 # 46

8–68 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x\to -\infty} x \ln\left(1 - \frac{1}{x}\right)$$

[Solution]

This limit has the form $(-\infty)\cdot 0$.

$$\lim_{x\to -\infty} x\ln\left(1-\frac{1}{x}\right) = \lim_{x\to -\infty} \frac{\ln\left(1-\dfrac{1}{x}\right)}{\dfrac{1}{x}} \overset{\text{H}}{=} \lim_{x\to -\infty} \frac{\dfrac{1}{1-1/x}\cdot\dfrac{1}{x^2}}{-\dfrac{1}{x^2}} = \lim_{x\to -\infty} \frac{-1}{1-\dfrac{1}{x}} = \frac{-1}{1} = -1$$

Sec.6.8 # 53

8–68 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$$

[Solution]

This limit has the form $\infty-\infty$.

$$\lim_{x\to 0+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)=\lim_{x\to 0+}\frac{e^x-1-x}{x(e^x-1)}\overset{\text{H}}{=}\lim_{x\to 0+}\frac{e^x-1}{xe^x+e^x-1}\overset{\text{H}}{=}\lim_{x\to 0+}\frac{e^x}{xe^x+e^x+e^x}=\frac{1}{0+1+1}=\frac{1}{2}$$

Sec.6.8 # 62

8–68 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x\to\infty} x^{(\ln 2)/(1+\ln x)}$$

[Solution]

$$y=x^{(\ln 2)/(1+\ln x)} \quad\Rightarrow\quad \ln y=\frac{\ln 2}{1+\ln x}\ln x \quad\Rightarrow$$

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{(\ln 2)(\ln x)}{1+\ln x}\overset{\text{H}}{=}\lim_{x\to\infty}\frac{(\ln 2)(1/x)}{1/x}=\lim_{x\to\infty}\ln 2=\ln 2,\text{ so }\lim_{x\to\infty}x^{(\ln 2)/(1+\ln x)}=\lim_{x\to\infty}e^{\ln y}=e^{\ln 2}=2.$$

Sec.6.8 # 67

8–68 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x\to 0^+}(1+\sin 3x)^{1/x}$$

[Solution]

$$y=(1+\sin 3x)^{1/x} \quad\Rightarrow\quad \ln y=\frac{1}{x}\ln(1+\sin 3x) \quad\Rightarrow$$

$$\lim_{x\to 0+}\ln y=\lim_{x\to 0+}\frac{\ln(1+\sin 3x)}{x}\overset{\text{H}}{=}\lim_{x\to 0+}\frac{[1/(1+\sin 3x)]\cdot 3\cos 3x}{1}=\lim_{x\to 0+}\frac{3\cos 3x}{1+\sin 3x}=\frac{3\cdot 1}{1+0}=3 \quad\Rightarrow$$

$$\lim_{x\to 0+}(1+\sin 3x)^{1/x}=\lim_{x\to 0+}e^{\ln y}=e^3$$

Sec.7.1 # 7

Evaluate the integral.

$$\int (x^2 + 2x) \cos x \, dx$$

[Solution]

First let $u = x^2 + 2x$, $dv = \cos x \, dx \quad \Rightarrow \quad du = (2x + 2) \, dx$, $v = \sin x$. Then by Equation 2,

$I = \int (x^2 + 2x) \cos x \, dx = (x^2 + 2x) \sin x - \int (2x + 2) \sin x \, dx$. Next let $U = 2x + 2$, $dV = \sin x \, dx \quad \Rightarrow \quad dU = 2 \, dx$,

$V = -\cos x$, so $\int (2x + 2) \sin x \, dx = -(2x + 2) \cos x - \int -2 \cos x \, dx = -(2x + 2) \cos x + 2 \sin x$. Thus,

$I = (x^2 + 2x) \sin x + (2x + 2) \cos x - 2 \sin x + C$.

Sec.7.1 # 10

Evaluate the integral.

$$\int \ln \sqrt{x} \, dx$$

[Solution]

Let $u = \ln \sqrt{x}$, $dv = dx \quad \Rightarrow \quad du = \dfrac{1}{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} \, dx = \dfrac{1}{2x} \, dx$, $v = x$. Then by Equation 2,

$$\int \ln \sqrt{x} \, dx = x \ln \sqrt{x} - \int x \cdot \frac{1}{2x} \, dx = x \ln \sqrt{x} - \int \frac{1}{2} \, dx = x \ln \sqrt{x} - \frac{1}{2} x + C.$$

Note: We could start by using $\ln \sqrt{x} = \frac{1}{2} \ln x$.

Sec.7.1 # 13

Evaluate the integral.

$$\int t \csc^2 t \, dt$$

[Solution]

Let $u = t$, $dv = \csc^2 t \, dt \quad \Rightarrow \quad du = dt$, $v = -\cot t$. Then by Equation 2,

$$\int t \csc^2 t \, dt = -t \cot t - \int -\cot t \, dt = -t \cot t + \int \frac{\cos t}{\sin t} \, dt = -t \cot t + \int \frac{1}{z} \, dz \qquad \begin{bmatrix} z = \sin t, \\ dz = \cos t \, dt \end{bmatrix}$$

$$= -t \cot t + \ln |z| + C = -t \cot t + \ln |\sin t| + C$$

Sec.7.1 # 19

Evaluate the integral.

$\int z^3 e^z \, dz$

[Solution]

First let $u = z^3$, $dv = e^z dz \quad \Rightarrow \quad du = 3z^2 dz$, $v = e^z$. Then $I_1 = \int z^3 e^z dz = z^3 e^z - 3 \int z^2 e^z dz$. Next let $u_1 = z^2$, $dv_1 = e^z dz \quad \Rightarrow \quad du_1 = 2z\, dz$, $v_1 = e^z$. Then $I_2 = z^2 e^z - 2 \int z e^z dz$. Finally, let $u_2 = z$, $dv_2 = e^z dz \quad \Rightarrow \quad du_2 = dz$, $v_2 = e^z$. Then $\int z e^z dz = z e^z - \int e^z dz = z e^z - e^z + C_1$. Substituting in the expression for I_2, we get $I_2 = z^2 e^z - 2(z e^z - e^z + C_1) = z^2 e^z - 2z e^z + 2e^z - 2C_1$. Substituting the last expression for I_2 into I_1 gives $I_1 = z^3 e^z - 3(z^2 e^z - 2z e^z + 2e^z - 2C_1) = z^3 e^z - 3z^2 e^z + 6z e^z - 6e^z + C$, where $C = 6C_1$.