

Calculus(II) Quiz3(03/26)

1.

Evaluate the indefinite integral.

$$\int \sqrt{\cot x} \csc^2 x \, dx$$

[Solution]

Let  $u = \cot x$ . Then  $du = -\csc^2 x \, dx$  and  $\csc^2 x \, dx = -du$ , so

$$\int \sqrt{\cot x} \csc^2 x \, dx = \int \sqrt{u} (-du) = -\frac{u^{3/2}}{3/2} + C = -\frac{2}{3}(\cot x)^{3/2} + C.$$

2.

Evaluate the definite integral.

$$\int_1^2 x \sqrt{x-1} \, dx$$

[Solution]

Let  $u = x - 1$ , so  $u + 1 = x$  and  $du = dx$ . When  $x = 1$ ,  $u = 0$ ; when  $x = 2$ ,  $u = 1$ . Thus,

$$\int_1^2 x \sqrt{x-1} \, dx = \int_0^1 (u+1)\sqrt{u} \, du = \int_0^1 (u^{3/2} + u^{1/2}) \, du = \left[ \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} \right]_0^1 = \frac{2}{5} + \frac{2}{3} = \frac{16}{15}.$$