Calculus(II) HW2 (3/5)

Sec. 4.2 # 22

Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

$$\int_1^4 (x^2 + 2x - 5)\,dx$$

[Solution]

$$\int_1^4 (x^2+2x-5)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \qquad [\Delta x = 3/n \text{ and } x_i = 1+3i/n]$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(1+\frac{3i}{n}\right)^2 + 2\left(1+\frac{3i}{n}\right) - 5\right]\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[\sum_{i=1}^{n}\left(1+\frac{6i}{n}+\frac{9i^2}{n^2}+2+\frac{6i}{n}-5\right)\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[\sum_{i=1}^{n}\left(\frac{9}{n^2}\cdot i^2+\frac{12}{n}\cdot i-2\right)\right] = \lim_{n\to\infty}\frac{3}{n}\left[\frac{9}{n^2}\sum_{i=1}^{n} i^2+\frac{12}{n}\sum_{i=1}^{n} i-\sum_{i=1}^{n} 2\right]$$

$$= \lim_{n\to\infty}\left(\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{36}{n^2}\cdot\frac{n(n+1)}{2}-\frac{6}{n}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\frac{9}{2}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}+18\cdot\frac{n+1}{n}-6\right)$$

$$= \lim_{n\to\infty}\left[\frac{9}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+18\left(1+\frac{1}{n}\right)-6\right] = \tfrac{9}{2}\cdot 1\cdot 2+18\cdot 1-6 = 21$$

Sec. 4.2 # 29

Express the integral as a limit of Riemann sums. Do not evaluate the limit.

$$\int_1^3 \sqrt{4+x^2}\,dx$$

[Solution]

$f(x)=\sqrt{4+x^2}$, $a=1$, $b=3$, and $\Delta x = \dfrac{3-1}{n} = \dfrac{2}{n}$. Using Theorem 4, we get $x_i^* = x_i = 1 + i\,\Delta x = 1+\dfrac{2i}{n}$, so

$$\int_1^3 \sqrt{4+x^2}\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{4+\left(1+\frac{2i}{n}\right)^2}\cdot\frac{2}{n}.$$

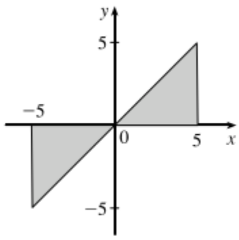
Sec. 4.2 # 38

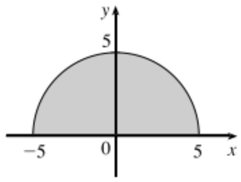
Evaluate the integral by interpreting it in terms of areas.

$$\int_{-5}^{5} \left(x - \sqrt{25 - x^2}\right) dx$$

[Solution]

$\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx = \int_{-5}^{5} x\,dx - \int_{-5}^{5}\sqrt{25-x^2}\,dx$. By symmetry, the value of the first integral is 0 since the shaded area above the x-axis equals the shaded area below the x-axis. The second integral can be interpreted as one half the area of a circle with radius 5; that is, $\frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi$. Thus, the value of the original integral is $0 - \frac{25}{2}\pi = -\frac{25}{2}\pi$.





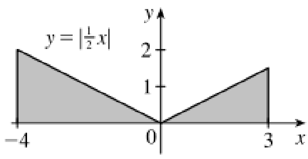
Sec. 4.2 # 39

Evaluate the integral by interpreting it in terms of areas.

$$\int_{-4}^{3} \left|\tfrac{1}{2}x\right| dx$$

[Solution]

$\int_{-4}^{3}\left|\frac{1}{2}x\right|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $\frac{1}{2}(4)(2) + \frac{1}{2}(3)\left(\frac{3}{2}\right) = 4 + \frac{9}{4} = \frac{25}{4}$.



Sec. 4.2 # 48

If $\int_2^8 f(x)\,dx = 7.3$ and $\int_2^4 f(x)\,dx = 5.9$, find $\int_4^8 f(x)\,dx$.

[Solution]

$\int_2^4 f(x)\,dx + \int_4^8 f(x)\,dx = \int_2^8 f(x)\,dx$, so $\int_4^8 f(x)\,dx = \int_2^8 f(x)\,dx - \int_2^4 f(x)\,dx = 7.3 - 5.9 = 1.4$.

Sec. 4.2 # 57

Use the properties of integrals to verify the inequality without evaluating the integrals

$$2 \leqslant \int_{-1}^{1} \sqrt{1 + x^2}\,dx \leqslant 2\sqrt{2}$$

[Solution]

If $-1 \le x \le 1$, then $0 \le x^2 \le 1$ and $1 \le 1 + x^2 \le 2$, so $1 \le \sqrt{1+x^2} \le \sqrt{2}$ and

$1[1-(-1)] \le \int_{-1}^{1}\sqrt{1+x^2}\,dx \le \sqrt{2}\,[1-(-1)]$ [Property 8]; that is, $2 \le \int_{-1}^{1}\sqrt{1+x^2}\,dx \le 2\sqrt{2}$.

Sec. 4.2 # 61

Use Property 8 of integrals to estimate the value of the integral.

$$\int_0^2 \frac{1}{1+x^2}\,dx$$

[Solution]

If $0 \le x \le 2$, then $1 \le 1+x^2 \le 5$ and $\frac{1}{5} \le \frac{1}{1+x^2} \le 1$, so $\frac{1}{5}(2-0) \le \int_0^2 \frac{1}{1+x^2}\,dx \le 1(2-0)$;

that is, $\frac{2}{5} \le \int_0^2 \frac{1}{1+x^2}\,dx \le 2$.

Sec. 4.3 # 9

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$g(x) = \int_1^x \frac{1}{t^3+1}\,dt$$

[Solution]

$f(t) = \frac{1}{t^3+1}$ and $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$, so by FTC1, $g'(x) = f(x) = \frac{1}{x^3+1}$. Note that the lower limit, 1, could be any real number greater than -1 and not affect this answer.

Sec. 4.3 # 14

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$G(x) = \int_x^1 \cos\sqrt{t}\,dt$$

[Solution]

$$G(x) = \int_x^1 \cos\sqrt{t}\,dt = -\int_1^x \cos\sqrt{t}\,dt \quad \Rightarrow \quad G'(x) = -\frac{d}{dx}\int_1^x \cos\sqrt{t}\,dt = -\cos\sqrt{x}$$