Calculus(II) HW8 (05/03)

Sec.6.2* # 3

Use the Laws of Logarithms to expand the quantity.

$\ln \frac{x^2}{y^3 z^4}$

[Solution]

$$\ln \frac{x^2}{y^3 z^4} = \ln x^2 - \ln(y^3 z^4) = 2\ln x - (\ln y^3 + \ln z^4) = 2\ln x - 3\ln y - 4\ln z$$

Sec.6.2* # 4

Use the Laws of Logarithms to expand the quantity.

$\ln\left(s^4 \sqrt{t\sqrt{u}}\right)$

[Solution]

$$\ln\left(s^4\sqrt{t\sqrt{u}}\right) = \ln s^4 (tu^{1/2})^{1/2} = \ln s^4 t^{1/2} u^{1/4} = \ln s^4 + \ln t^{1/2} + \ln u^{1/4} = 4\ln s + \tfrac{1}{2}\ln t + \tfrac{1}{4}\ln u$$

Sec.6.2* # 9

Express the quantity as a single logarithm.

$\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}[\ln x - \ln(x^2+3x+2)^2]$

[Solution]

$$\begin{aligned}
\tfrac{1}{3}\ln(x+2)^3 + \tfrac{1}{2}\left[\ln x - \ln(x^2+3x+2)^2\right] &= \ln[(x+2)^3]^{1/3} + \tfrac{1}{2}\ln\frac{x}{(x^2+3x+2)^2} && \text{[by Laws 3, 2]} \\
&= \ln(x+2) + \ln\frac{\sqrt{x}}{x^2+3x+2} && \text{[by Law 3]} \\
&= \ln\frac{(x+2)\sqrt{x}}{(x+1)(x+2)} && \text{[by Law 1]} \\
&= \ln\frac{\sqrt{x}}{x+1}
\end{aligned}$$

Note that since $\ln x$ is defined for $x > 0$, we have $x+1$, $x+2$, and x^2+3x+2 all positive, and hence their logarithms are defined.

Sec.6.2* # 10

Express the quantity as a single logarithm.

$\ln b + 2\ln c - 3\ln d$

[Solution]

$$\begin{aligned} \ln b + 2\ln c - 3\ln d &= \ln b + \ln c^2 - \ln d^3 && \text{[by Law 3]} \\ &= \ln bc^2 - \ln d^3 && \text{[by Law 1]} \\ &= \ln \frac{bc^2}{d^3} && \text{[by Law 2]} \end{aligned}$$

Sec.6.2* # 16

> Find the limit.
>
> $\lim_{x\to\infty} [\ln(2+x) - \ln(1+x)]$

[Solution]

$$\lim_{x\to\infty} [\ln(2+x) - \ln(1+x)] = \lim_{x\to\infty} \ln\left(\frac{2+x}{1+x}\right) = \lim_{x\to\infty} \ln\left(\frac{2/x+1}{1/x+1}\right) = \ln\frac{1}{1} = \ln 1 = 0$$

Sec.6.2* # 20

> Differentiate the function.
>
> $f(x) = \ln(\sin^2 x)$

[Solution]

$$f(x) = \ln(\sin^2 x) = \ln(\sin x)^2 = 2\ln|\sin x| \quad \Rightarrow \quad f'(x) = 2\cdot\frac{1}{\sin x}\cdot\cos x = 2\cot x$$

Sec.6.2* # 23

> Differentiate the function.
>
> $f(x) = \sin x \ln(5x)$

[Solution]

$$f(x) = \sin x \ln(5x) \quad \Rightarrow \quad f'(x) = \sin x\cdot\frac{1}{5x}\cdot\frac{d}{dx}(5x) + \ln(5x)\cdot\cos x = \frac{\sin x\cdot 5}{5x} + \cos x \ln(5x) = \frac{\sin x}{x} + \cos x \ln(5x)$$

Sec.6.2* # 27

> Differentiate the function.
>
> $G(y) = \ln\dfrac{(2y+1)^5}{\sqrt{y^2+1}}$

[Solution]

$$G(y) = \ln\frac{(2y+1)^5}{\sqrt{y^2+1}} = \ln(2y+1)^5 - \ln(y^2+1)^{1/2} = 5\ln(2y+1) - \tfrac{1}{2}\ln(y^2+1) \quad \Rightarrow$$

$$G'(y) = 5\cdot\frac{1}{2y+1}\cdot 2 - \frac{1}{2}\cdot\frac{1}{y^2+1}\cdot 2y = \frac{10}{2y+1} - \frac{y}{y^2+1} \quad \left[\text{or } \frac{8y^2-y+10}{(2y+1)(y^2+1)}\right]$$

Sec.6.2* # 29

> Differentiate the function.
>
> $F(t) = (\ln t)^2 \sin t$

[Solution]

$$F(t) = (\ln t)^2 \sin t \quad \Rightarrow \quad F'(t) = (\ln t)^2 \cos t + \sin t \cdot 2\ln t \cdot \frac{1}{t} = \ln t \left(\ln t \, \cos t + \frac{2\sin t}{t} \right)$$

Sec.6.2* # 36

Differentiate the function.

$y = \ln(\csc x - \cot x)$

[Solution]

$y = \ln(\csc x - \cot x) \quad \Rightarrow$

$$y' = \frac{1}{\csc x - \cot x} \frac{d}{dx}(\csc x - \cot x) = \frac{1}{\csc x - \cot x}(-\csc x \, \cot x + \csc^2 x) = \frac{\csc x(\csc x - \cot x)}{\csc x - \cot x} = \csc x$$

Sec.6.2* # 42

Differentiate f and find the domain of f.

$f(x) = \ln \ln \ln x$

[Solution]

$$f(x) = \ln \ln \ln x \quad \Rightarrow \quad f'(x) = \frac{1}{\ln \ln x} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}.$$

$\text{Dom}(f) = \{x \mid \ln \ln x > 0\} = \{x \mid \ln x > 1\} = \{x \mid x > e\} = (e, \infty).$

Sec.6.2* # 57

Discuss the curve under the guidelines of Section 3.5.

$y = \ln(1 + x^2)$

[Solution]

$y = f(x) = \ln(1 + x^2)$ **A.** $D = \mathbb{R}$ **B.** Both intercepts are 0. **C.** $f(-x) = f(x)$, so the curve is symmetric about the y-axis. **D.** $\lim_{x \to \pm\infty} \ln(1 + x^2) = \infty$, no asymptotes. **E.** $f'(x) = \dfrac{2x}{1 + x^2} > 0 \iff$ $x > 0$, so f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

F. $f(0) = 0$ is a local and absolute minimum.

G. $f''(x) = \dfrac{2(1 + x^2) - 2x(2x)}{(1 + x^2)^2} = \dfrac{2(1 - x^2)}{(1 + x^2)^2} > 0 \iff$

$|x| < 1$, so f is CU on $(-1, 1)$, CD on $(-\infty, -1)$ and $(1, \infty)$.

IP $(1, \ln 2)$ and $(-1, \ln 2)$.

H.