Calculus(II) Quiz4(04/09)

1.

13–28 Sketch the region enclosed by the given curves and find its area.

$y = \cos \pi x, \quad y = 4x^2 - 1$

[Solution]

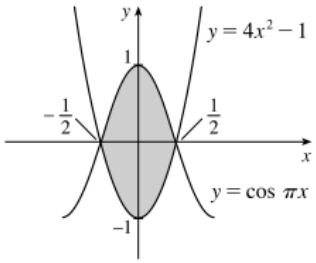
By inspection, the curves intersect at $x = \pm\frac{1}{2}$.

$$A = \int_{-1/2}^{1/2} [\cos \pi x - (4x^2 - 1)]\, dx$$

$$= 2\int_{0}^{1/2} (\cos \pi x - 4x^2 + 1)\, dx \quad \text{[by symmetry]}$$

$$= 2\left[\tfrac{1}{\pi}\sin \pi x - \tfrac{4}{3}x^3 + x\right]_0^{1/2} = 2\left[\left(\tfrac{1}{\pi} - \tfrac{1}{6} + \tfrac{1}{2}\right) - 0\right]$$

$$= 2\left(\tfrac{1}{\pi} + \tfrac{1}{3}\right) = \tfrac{2}{\pi} + \tfrac{2}{3}$$



2.

1–18 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

$y = x^2, \; x = y^2; \quad \text{about } y = 1$

[Solution]

A cross-section is a washer with inner radius $1 - \sqrt{x}$ and outer radius $1 - x^2$, so its area is

$$A(x) = \pi\left[(1 - x^2)^2 - (1 - \sqrt{x}\,)^2\right]$$

$$= \pi\left[(1 - 2x^2 + x^4) - (1 - 2\sqrt{x} + x)\right]$$

$$= \pi\left(x^4 - 2x^2 + 2\sqrt{x} - x\right).$$

$$V = \int_0^1 A(x)\, dx = \int_0^1 \pi(x^4 - 2x^2 + 2x^{1/2} - x)\, dx$$

$$= \pi\left[\tfrac{1}{5}x^5 - \tfrac{2}{3}x^3 + \tfrac{4}{3}x^{3/2} - \tfrac{1}{2}x^2\right]_0^1$$

$$= \pi\left(\tfrac{1}{5} - \tfrac{2}{3} + \tfrac{4}{3} - \tfrac{1}{2}\right) = \tfrac{11}{30}\pi$$