

Calculus(II) Quiz7(05/14)

1.

61-64 Use logarithmic differentiation to find the derivative of the function.

$$y = \sqrt{\frac{x-1}{x^4+1}}$$

[Solution]

$$y = \sqrt{\frac{x-1}{x^4+1}} \Rightarrow \ln y = \ln\left(\frac{x-1}{x^4+1}\right)^{1/2} \Rightarrow \ln y = \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x^4+1) \Rightarrow$$
$$\frac{1}{y}y' = \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\frac{1}{x^4+1} \cdot 4x^3 \Rightarrow y' = y\left(\frac{1}{2(x-1)} - \frac{2x^3}{x^4+1}\right) \Rightarrow y' = \sqrt{\frac{x-1}{x^4+1}}\left(\frac{1}{2x-2} - \frac{2x^3}{x^4+1}\right)$$

2.

Solve each equation for x .

$$\ln x + \ln(x-1) = 1$$

[Solution]

$\ln x + \ln(x-1) = \ln(x(x-1)) = 1 \Leftrightarrow x(x-1) = e^1 \Leftrightarrow x^2 - x - e = 0$. The quadratic formula (with $a = 1$, $b = -1$, and $c = -e$) gives $x = \frac{1}{2}(1 \pm \sqrt{1+4e})$, but we reject the negative root since the natural logarithm is not defined for $x < 0$. So $x = \frac{1}{2}(1 + \sqrt{1+4e})$.