

Calculus(II) Quiz2(03/19)

1.

Evaluate the integral.

$$\int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$$

[Solution]

$$\begin{aligned} \int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta &= \int_0^{\pi/4} \left(\frac{1}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} \right) d\theta = \int_0^{\pi/4} (\sec^2 \theta + 1) d\theta \\ &= [\tan \theta + \theta]_0^{\pi/4} = \left(\tan \frac{\pi}{4} + \frac{\pi}{4} \right) - (0 + 0) = 1 + \frac{\pi}{4} \end{aligned}$$

2.

Find the derivative of the function.

$$h(x) = \int_{\sqrt{x}}^{x^3} \cos(t^2) dt$$

[Solution]

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$$h(x) = \int_{\sqrt{x}}^{x^3} \cos(t^2) dt = \int_{\sqrt{x}}^0 \cos(t^2) dt + \int_0^{x^3} \cos(t^2) dt = - \int_0^{\sqrt{x}} \cos(t^2) dt + \int_0^{x^3} \cos(t^2) dt \Rightarrow$$

$$h'(x) = -\cos\left((\sqrt{x})^2\right) \cdot \frac{d}{dx}(\sqrt{x}) + [\cos(x^3)^2] \cdot \frac{d}{dx}(x^3) = -\frac{1}{2\sqrt{x}} \cos x + 3x^2 \cos(x^6)$$