Calculus(II) HW1 (2/26)

Sec. 4.1 # 21

21–23 Use Definition 2 to find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$$f(x) = \frac{2x}{x^2+1}, \quad 1 \leqslant x \leqslant 3$$

[Solution]

$f(x) = \dfrac{2x}{x^2+1}$, $1 \le x \le 3$. $\Delta x = (3-1)/n = 2/n$ and $x_i = 1 + i\Delta x = 1 + 2i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2(1+2i/n)}{(1+2i/n)^2+1} \cdot \frac{2}{n}.$$

Sec. 4.1 # 25

24–25 Determine a region whose area is equal to the given limit. Do not evaluate the limit.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan \frac{i\pi}{4n}$$

[Solution]

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan \frac{i\pi}{4n}$ can be interpreted as the area of the region lying under the graph of $y = \tan x$ on the interval $[0, \frac{\pi}{4}]$, since for $y = \tan x$ on $[0, \frac{\pi}{4}]$ with $\Delta x = \dfrac{\pi/4 - 0}{n} = \dfrac{\pi}{4n}$, $x_i = 0 + i\,\Delta x = \dfrac{i\pi}{4n}$, and $x_i^* = x_i$, the expression for the area is $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \tan\left(\frac{i\pi}{4n}\right) \frac{\pi}{4n}$. Note that this answer is not unique, since the expression for the area is the same for the function $y = \tan(x - k\pi)$ on the interval $[k\pi, k\pi + \frac{\pi}{4}]$, where k is any integer.

Sec. 4.2 # 18

17–20 Express the limit as a definite integral on the given interval.

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i\sqrt{1+x_i^3}\,\Delta x, \quad [2, 5]$$

[Solution]

On $[2, 5]$, $\lim_{n\to\infty} \sum_{i=1}^{n} x_i\sqrt{1+x_i^3}\,\Delta x = \int_2^5 x\sqrt{1+x^3}\,dx.$

Sec. 4.2 # 20

17–20 Express the limit as a definite integral on the given interval.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i} \Delta x, \quad [\pi, 2\pi]$$

[Solution]

On $[\pi, 2\pi]$, $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i} \Delta x = \int_{\pi}^{2\pi} \frac{\cos x}{x}\, dx.$