Calculus(II) Quiz10(06/04)

1.

Evaluate the integral.

$$\int \tan^{-1} 2y \, dy$$

[Solution]

Let $u = \tan^{-1} 2y$, $dv = dy \quad \Rightarrow \quad du = \dfrac{2}{1+4y^2}\,dy$, $v = y$. Then by Equation 2,

$$\int \tan^{-1} 2y\,dy = y\tan^{-1} 2y - \int \frac{2y}{1+4y^2}\,dy = y\tan^{-1} 2y - \int \frac{1}{t}\left(\frac{1}{4}\,dt\right) \qquad \begin{bmatrix} t = 1+4y^2, \\ dt = 8y\,dy \end{bmatrix}$$

$$= y\tan^{-1} 2y - \tfrac{1}{4}\ln|t| + C = y\tan^{-1} 2y - \tfrac{1}{4}\ln(1+4y^2) + C$$

2.

8–68 Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1}$$

[Solution]

$$y = \left(\frac{2x-3}{2x+5}\right)^{2x+1} \quad \Rightarrow \quad \ln y = (2x+1)\,\ln\left(\frac{2x-3}{2x+5}\right) \quad \Rightarrow$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(2x-3)-\ln(2x+5)}{1/(2x+1)} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{2/(2x-3)-2/(2x+5)}{-2/(2x+1)^2} = \lim_{x\to\infty} \frac{-8(2x+1)^2}{(2x-3)(2x+5)}$$

$$= \lim_{x\to\infty} \frac{-8(2+1/x)^2}{(2-3/x)(2+5/x)} = -8 \quad \Rightarrow \quad \lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1} = e^{-8}$$