Calculus(II) HW10 (05/14)

Sec.6.3* # 35

33–52 Differentiate the function.
$f(x) = (3x^2 - 5x)e^x$

[Solution]

By the Product Rule, $f(x) = (3x^2 - 5x)e^x \quad \Rightarrow$

$$f'(x) = (3x^2 - 5x)(e^x)' + e^x(3x^2 - 5x)' = (3x^2 - 5x)e^x + e^x(6x - 5)$$
$$= e^x[(3x^2 - 5x) + (6x - 5)] = e^x(3x^2 + x - 5)$$

Sec.6.3* # 38

33–52 Differentiate the function.
$g(x) = e^{x^2 - x}$

[Solution]

$$g(x) = e^{x^2 - x} \quad \Rightarrow \quad g'(x) = e^{x^2 - x}(2x - 1)$$

Sec.6.3* # 41

33–52 Differentiate the function.
$f(x) = \dfrac{x^2 e^x}{x^2 + e^x}$

[Solution]

$$f(x) = \frac{x^2 e^x}{x^2 + e^x} \quad \overset{\text{QR}}{\Rightarrow}$$

$$f'(x) = \frac{(x^2 + e^x)\left[x^2 e^x + e^x(2x)\right] - x^2 e^x(2x + e^x)}{(x^2 + e^x)^2} = \frac{x^4 e^x + 2x^3 e^x + x^2 e^{2x} + 2xe^{2x} - 2x^3 e^x - x^2 e^{2x}}{(x^2 + e^x)^2}$$

$$= \frac{x^4 e^x + 2xe^{2x}}{(x^2 + e^x)^2} = \frac{xe^x(x^3 + 2e^x)}{(x^2 + e^x)^2}$$

Sec.6.3* # 44

33–52 Differentiate the function.

$f(t) = \tan(1 + e^{2t})$

[Solution]

$$f(t) = \tan(1+e^{2t}) \quad \Rightarrow \quad f'(t) = \sec^2(1+e^{2t}) \cdot (1+e^{2t})' = 2e^{2t}\sec^2(1+e^{2t})$$

Sec.6.3* # 52

33–52 Differentiate the function.

$f(t) = e^{k\tan\sqrt{x}}$

[Solution]

$$y = e^{k\tan\sqrt{x}} \quad \Rightarrow \quad y' = e^{k\tan\sqrt{x}} \cdot \frac{d}{dx}\left(k\tan\sqrt{x}\right) = e^{k\tan\sqrt{x}}\left(k\sec^2\sqrt{x} \cdot \tfrac{1}{2}x^{-1/2}\right) = \frac{k\sec^2\sqrt{x}}{2\sqrt{x}}e^{k\tan\sqrt{x}}$$

Sec.6.3* # 53

53–54 Find an equation of the tangent line to the curve at the given point.

$y = e^{2x}\cos\pi x, \quad (0, 1)$

[Solution]

$$y = e^{2x}\,\cos\pi x \quad \Rightarrow \quad y' = e^{2x}(-\pi\sin\pi x) + (\cos\pi x)(2e^{2x}) = e^{2x}(2\cos\pi x - \pi\sin\pi x).$$

At $(0, 1)$, $y' = 1(2-0) = 2$, so an equation of the tangent line is $y - 1 = 2(x-0)$, or $y = 2x + 1$.

Sec.6.3* # 70

69–70 Find the absolute maximum and absolute minimum values of f on the given interval.

$f(x) = xe^{x/2}, \quad [-3, 1]$

[Solution]

$f(x) = xe^{x/2}$, $[-3, 1]$. $f'(x) = xe^{x/2}\left(\frac{1}{2}\right) + e^{x/2}(1) = e^{x/2}\left(\frac{1}{2}x + 1\right)$. $f'(x) = 0 \Leftrightarrow \frac{1}{2}x + 1 = 0 \Leftrightarrow x = -2$.

$f(-3) = -3e^{-3/2} \approx -0.669$, $f(-2) = -2e^{-1} \approx -0.736$, and $f(1) = e^{1/2} \approx 1.649$. So $f(1) = e^{1/2}$ is the absolute maximum value and $f(-2) = -2/e$ is the absolute minimum value.

Sec.6.3* # 73

Discuss the curve using the guidelines of Section 3.5.

$y = e^{-1/(x+1)}$

[Solution]

$y = f(x) = e^{-1/(x+1)}$ **A.** $D = \{x \mid x \neq -1\} = (-\infty, -1) \cup (-1, \infty)$ **B.** No x-intercept; y-intercept $= f(0) = e^{-1}$

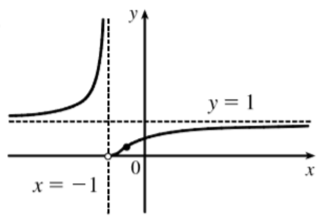
C. No symmetry **D.** $\lim_{x\to\pm\infty} e^{-1/(x+1)} = 1$ since $-1/(x+1) \to 0$, so $y = 1$ is a HA. $\lim_{x\to -1^+} e^{-1/(x+1)} = 0$ since $-1/(x+1) \to -\infty$, $\lim_{x\to -1^-} e^{-1/(x+1)} = \infty$ since $-1/(x+1) \to \infty$, so $x = -1$ is a VA.

E. $f'(x) = e^{-1/(x+1)}/(x+1)^2 \Rightarrow f'(x) > 0$ for all x except 1, so f is increasing on $(-\infty, -1)$ and $(-1, \infty)$. **F.** No extreme values

G. $f''(x) = \dfrac{e^{-1/(x+1)}}{(x+1)^4} + \dfrac{e^{-1/(x+1)}(-2)}{(x+1)^3} = -\dfrac{e^{-1/(x+1)}(2x+1)}{(x+1)^4} \Rightarrow$

$f''(x) > 0 \Leftrightarrow 2x + 1 < 0 \Leftrightarrow x < -\frac{1}{2}$, so f is CU on $(-\infty, -1)$ and $\left(-1, -\frac{1}{2}\right)$, and CD on $\left(-\frac{1}{2}, \infty\right)$. f has an IP at $\left(-\frac{1}{2}, e^{-2}\right)$.

H.



Sec.6.3* # 87

Evaluate the integral.

$\int e^x \sqrt{1 + e^x}\, dx$

[Solution]

Let $u = 1 + e^x$. Then $du = e^x\, dx$, so $\int e^x \sqrt{1 + e^x}\, dx = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1 + e^x)^{3/2} + C$.

Sec.6.3* # 90

Evaluate the integral. $\int e^x \cos(e^x)\, dx$

[Solution]

Let $u = e^x$. Then $du = e^x\, dx$, so $\int e^x \cos(e^x)\, dx = \int \cos u\, du = \sin u + C = \sin(e^x) + C$.

Sec.6.4* # 24

Find the limit. $\lim_{x \to 3^+} \log_{10}(x^2 - 5x + 6)$

[Solution]

Let $t = x^2 - 5x + 6$. As $x \to 3^+$, $t = (x-2)(x-3) \to 0^+$. $\lim_{x \to 3^+} \log_{10}(x^2 - 5x + 6) = \lim_{t \to 0^+} \log_{10} t = -\infty$

[analogous to (4) in Section 6.2*].

Sec.6.4* # 30

Differentiate the function. $G(u) = (1 + 10^{\ln u})^6$

[Solution]

$G(u) = (1 + 10^{\ln u})^6 \quad \Rightarrow$

$G'(u) = 6(1 + 10^{\ln u})^5 \frac{d}{du}(1 + 10^{\ln u}) = 6(1 + 10^{\ln u})^5\, 10^{\ln u}\, \ln 10 \frac{d}{du}(\ln u) = 6 \ln 10 (1 + 10^{\ln u})^5 \cdot 10^{\ln u}/u$

Sec.6.4* # 33

Differentiate the function. $y = x \log_4 \sin x$

[Solution]

$$y = x\log_4 \sin x \quad \Rightarrow \quad y' = x \cdot \frac{1}{\sin x \ln 4} \cdot \cos x + \log_4 \sin x \cdot 1 = \frac{x \cot x}{\ln 4} + \log_4 \sin x$$

Sec.6.4* # 41

> Differentiate the function.
>
> $y = (\tan x)^{1/x}$

[Solution]

$$y = (\tan x)^{1/x} \quad \Rightarrow \quad \ln y = \ln(\tan x)^{1/x} \quad \Rightarrow \quad \ln y = \frac{1}{x}\ln \tan x \quad \Rightarrow$$

$$\frac{1}{y}y' = \frac{1}{x} \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln \tan x \cdot \left(-\frac{1}{x^2}\right) \quad \Rightarrow \quad y' = y\left(\frac{\sec^2 x}{x \tan x} - \frac{\ln \tan x}{x^2}\right) \quad \Rightarrow$$

$$y' = (\tan x)^{1/x}\left(\frac{\sec^2 x}{x\tan x} - \frac{\ln \tan x}{x^2}\right) \quad \text{or} \quad y' = (\tan x)^{1/x} \cdot \frac{1}{x}\left(\csc x \sec x - \frac{\ln \tan x}{x}\right)$$

Sec.6.4* # 47

> Evaluate the integral.
>
> $\int \frac{\log_{10} x}{x}\, dx$

[Solution]

$\int \frac{\log_{10} x}{x}\,dx = \int \frac{(\ln x)/(\ln 10)}{x}\,dx = \frac{1}{\ln 10}\int \frac{\ln x}{x}\,dx$. Now put $u = \ln x$, so $du = \frac{1}{x}\,dx$, and the expression becomes

$\frac{1}{\ln 10}\int u\,du = \frac{1}{\ln 10}\left(\frac{1}{2}u^2 + C_1\right) = \frac{1}{2\ln 10}(\ln x)^2 + C.$