Calculus(II) HW7 (04/16)

Sec.6.1 # 40

Find $(f^{-1})'(a)$.
$f(x) = 2x^3 + 3x^2 + 7x + 4, \quad a = 4$

[Solution]

$f(0) = 4 \Rightarrow f^{-1}(4) = 0$, and $f(x) = 2x^3 + 3x^2 + 7x + 4 \Rightarrow f'(x) = 6x^2 + 6x + 7$ and $f'(0) = 7$.

Thus, $(f^{-1})'(4) = \dfrac{1}{f'(f^{-1}(4))} = \dfrac{1}{f'(0)} = \dfrac{1}{7}$.

Sec.6.1 # 41

Find $(f^{-1})'(a)$.
$f(x) = 3 + x^2 + \tan(\pi x/2), \quad -1 < x < 1, \quad a = 3$

[Solution]

$f(0) = 3 \Rightarrow f^{-1}(3) = 0$, and $f(x) = 3 + x^2 + \tan(\pi x/2) \Rightarrow f'(x) = 2x + \frac{\pi}{2}\sec^2(\pi x/2)$ and $f'(0) = \frac{\pi}{2} \cdot 1 = \frac{\pi}{2}$. Thus, $(f^{-1})'(3) = 1/f'(f^{-1}(3)) = 1/f'(0) = 2/\pi$.

Sec.6.1 # 42

Find $(f^{-1})'(a)$.
$f(x) = \sqrt{x^3 + 4x + 4}, \quad a = 3$

[Solution]

$f(1) = 3 \Rightarrow f^{-1}(3) = 1$, and $f(x) = \sqrt{x^3 + 4x + 4} \Rightarrow f'(x) = \dfrac{3x^2 + 4}{2\sqrt{x^3 + 4x + 4}}$ and $f'(1) = \dfrac{7}{6}$. Thus,

$(f^{-1})'(3) = \dfrac{1}{f'(f^{-1}(3))} = \dfrac{1}{f'(1)} = \dfrac{1}{7/6} = \dfrac{6}{7}$.