Chapter 6. 機械波 Mechanical Waves - Sound

1. Two kinds of waves:

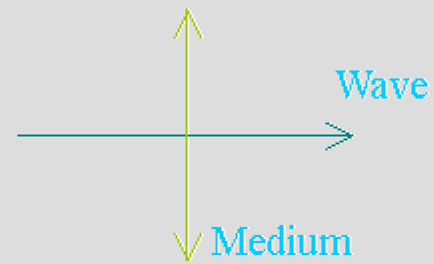
(1) With medium ⟶ Mechanical Wave 機械波

(2) Without medium ⟶ Electromagnetic Wave電磁波

2. Mechanical Wave:

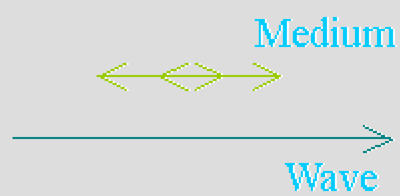
(1) Medium 振動方向 ⊥ **Wave** 進行方向

→ Transverse Wave 橫向波



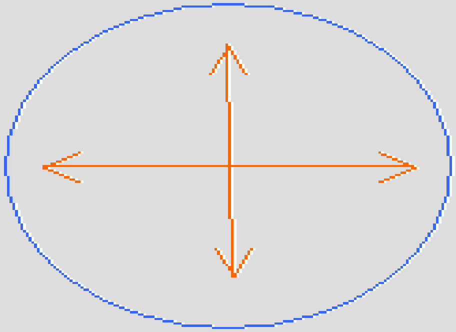
(2) Medium振動方向 ∥ **Wave**進行方向

→ Longitudinal Wave 縱向波



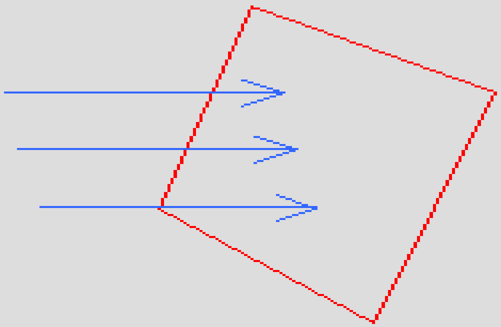
3、波前（Wavefront）⟹ 球面 Spherical Wave 球面波

ex：水波



4、波前 ⟹ 平面 Plane Wave 平面波

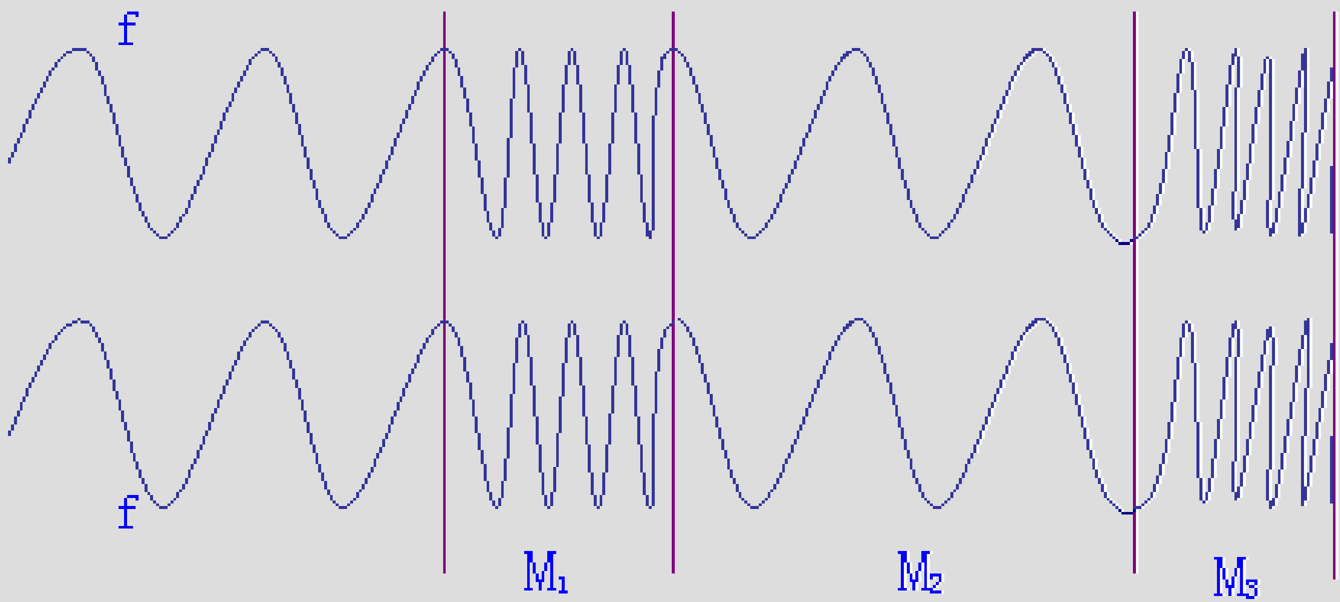
$$\vec{E} = \vec{E}_0 \sin\omega t$$



$$S = E \times B$$

$$\vec{B} = \vec{B}_0 \sin\omega t$$

5、f（波源頻率）經過不同介質 → 頻率不變



$V = f_s \cdot \lambda$ $f_1 = f_s$ $f_2 = f_s$ $f_3 = f_s$

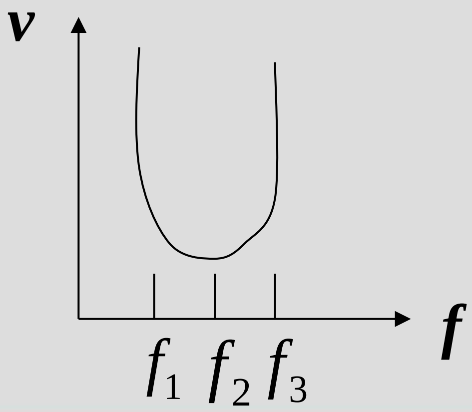
（不變）

6. 同頻率的波在不同介質中有規則的波速

橫向波 $v \propto \lambda \propto \frac{1}{\rho_{密度}}$

縱向波 $v \propto \rho_{密度}$

7. 不同頻率的波在同一介質中有規則的波速為散射現象(Dispersion)



波速大小不可由 f 決定,

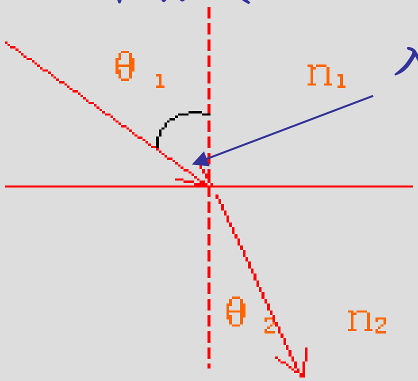
只可由 λ 或（介質密度）決定

8. 反射（Reflection）

疏→密 ⟶ 反射波反相

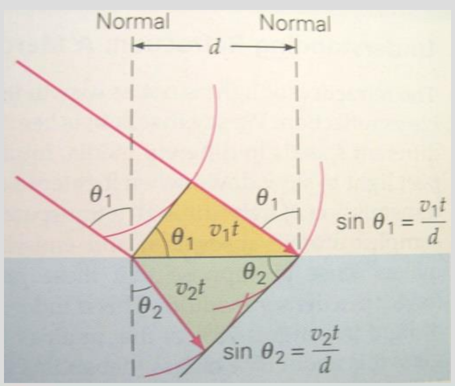
密→疏 ⟶ 反射波不反相

9. 折射（Refraction）



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

<<折射定律>>Refraction



$$\sin\theta_1 = \frac{v_1 t}{d} \quad \sin\theta_2 = \frac{v_2 t}{d}$$

$$\Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

$$\Rightarrow n = \frac{c}{v}$$

$$\Rightarrow n = \frac{c}{v} = \frac{\lambda f}{\lambda_m f}$$

$$\Rightarrow n = \frac{\lambda}{\lambda_m}$$

n: **index of refraction**

折射率

C: **light speed in vacuum**

（真空）

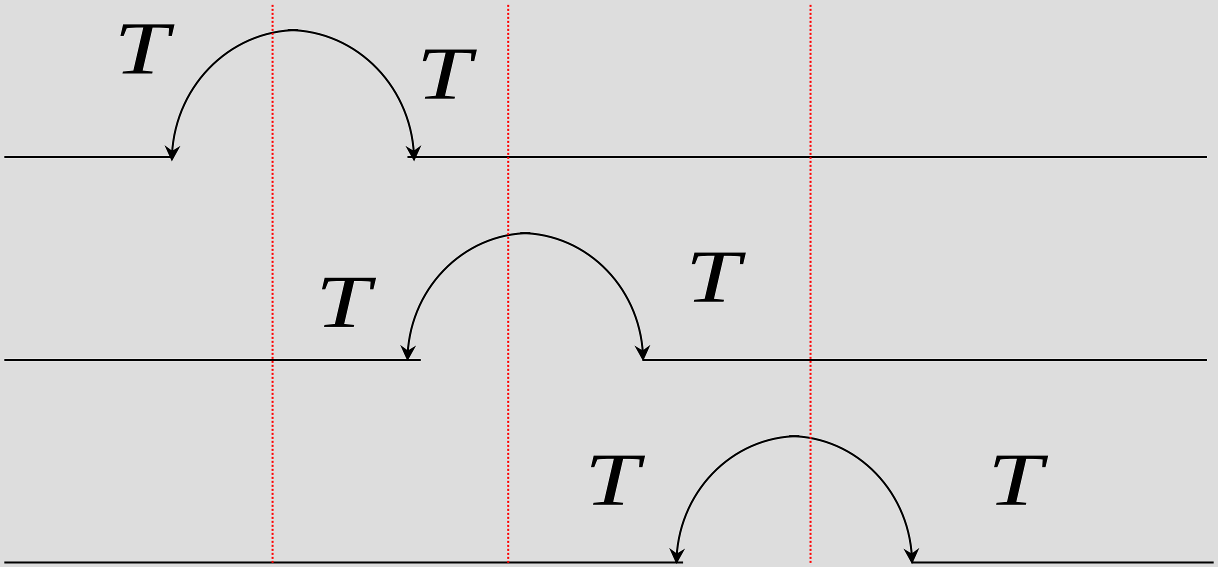
V: **light speed in medium**

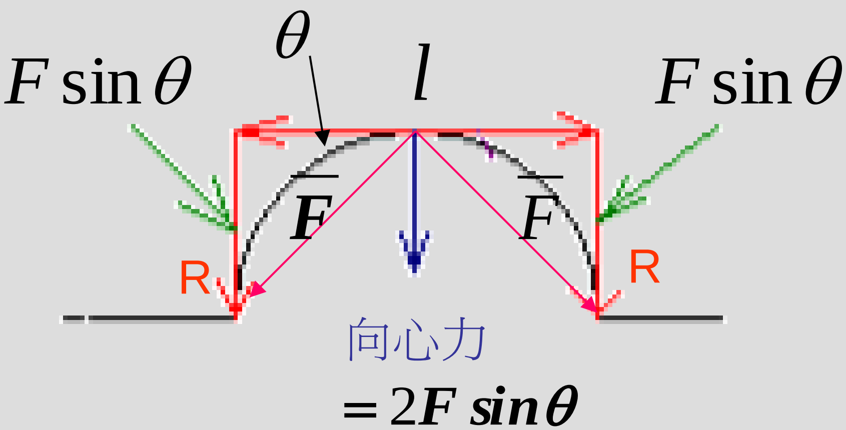
（介質）

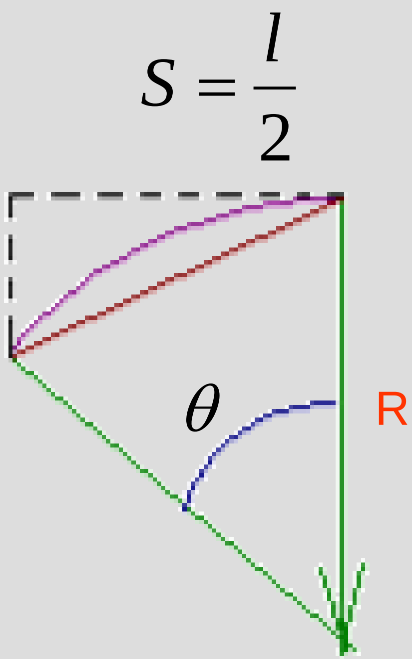
$$\Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

10. 繩波波速公式推導：<橫向波>

因繩波只有垂直振動並無水平移動，故繩索上任一點張力（Tension）相同。







$$R\theta = S \qquad \theta = \frac{S}{R} = \frac{\frac{l}{2}}{R}$$

$$2F\sin\theta = Ma = \frac{MV^2}{R} = \text{向心力}$$

is small, sin $\theta \approx \theta$, $R\theta = \frac{l}{2}$, $\theta = \frac{l}{2R}$ $\Rightarrow$ $F\frac{l}{R} = M\frac{V^2}{R}$

$$\because \mu = \left(\frac{M}{l}\right)$$ 一度空間質量密度

$$\therefore V(\text{波速}) = \left(\frac{F}{\mu}\right)^{\frac{1}{2}}$$

$$\Rightarrow V = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{\text{施力}}{\text{質量密度}}}$$

ATIS Lab.

Advanced Technology & Integrated Systems Laboratory

11. 音波波速公式推導: <縱向波>

→ 音波傳遞靠介質分子前後移動，移動速率可決定音波波速。

(分子結構越緊密，前後移動距離短，音波波速越快，$V_{固/液} > V_{氣}$)

$$\frac{1}{2}mV^2 = K_x = K_y = K_z = \frac{1}{2}mV_y^2 = \frac{1}{2}mV_z^2$$

→ **分子來回移動的動能來自周遭熱能 (溫度 $\rightarrow \frac{1}{2}K_BT$)**

$$K = \frac{1}{2}mV^2 = \frac{1}{2}mV_X^2 + \frac{1}{2}mV_y^2 + \frac{1}{2}mV_z^2 = K_x + K_y + K_z$$

$$= \frac{1}{2}K_BT + \frac{1}{2}K_BT + \frac{1}{2}K_BT = \frac{3}{2}K_BT$$

$$\Rightarrow V^2 = \frac{3K_BT}{m}, V_{r.m.s} = \sqrt{\frac{3K_BT}{m}}$$ （方均根速率）

$$V_{r.m.s} = \sqrt{\frac{3K_BT}{m}}$$ （介質分子來回移動速率）

$$\Rightarrow V_{SOUND} \propto V_{r.m.s}$$ （音波波速和分子來回移動速率成正比）

For Gas Molecules $\Rightarrow$ $V_{SOUND} = \sqrt{\frac{r}{3}}\, V_{r.m.s}$

對雙原子 Gas Molecules (O_2, N_2) $\Rightarrow$ $r = 1.4$

$$V_{SOUND} = \sqrt{\frac{1.4K_BT}{m}} \quad N_2 = 28 \bullet 4/5, O_2 = 32 \bullet 1/5, m = \frac{N_2 + O_2}{6.02 \times 10^{23}} \times \frac{1}{1000}$$

12、杜卜勒效應（Doppler Effect）是觀察者效應。

（V：波源波速；V_S：波源移動速度；V_0：觀察者移動速度；f_S：波源頻率不變

f_0：觀察到之頻率起伏隨〈A〉相對速度或〈B〉波長而改變）

〈A〉Stationary 靜止 Source, Moving Observer 觀察者 ($V_s=0$, $V_0\neq 0$)

⇒ 以觀察者靜止座標（觀察者只知自己移動），音源靜止不動，頻率變化

反應在波源波速 $V\pm V_0$ (相對速度)上。（觀察者認為λ波長並無改變）

$$\underline{V} = f_0 \underline{\lambda}$$

① ↓↑ ②

① V

② λ

⇒ 改變造成 f_0 高低的因素

- 當 V_0 趨近音源 → $V \rightarrow V + V_0 = V'$ 觀察者認為波速增加

$$\frac{V'}{\lambda} = f_0 = \frac{V + V_0}{\lambda} = \frac{V + V_0}{V/f_s} = f_s \frac{V + V_0}{V} = f_s\left(1 + \frac{V_0}{V}\right)$$ 頻率升高

- 當 V_0 遠離音源 → $V \rightarrow V - V_0 = V''$ 觀察者認為波速減少

$$\frac{V''}{\lambda} = f_0 = \frac{V - V_0}{\lambda} = \frac{V - V_0}{V/f_s} = f_s \frac{V - V_0}{V} = f_s\left(1 - \frac{V_0}{V}\right)$$ 頻率降低

<B> Moving Source，Stationary Observer

$$\left(V_S \neq 0, V_0 = 0\right)$$

以觀察者靜止座標來看(觀察者只知自己靜止），

(以無相對速度)音源靜止不動，頻率變化反應在

λ 波長 （觀察者認為波速不變）。

ΔS = 波源移動距離

V_s：波源移動速度

$$T = \frac{1}{f_s}$$

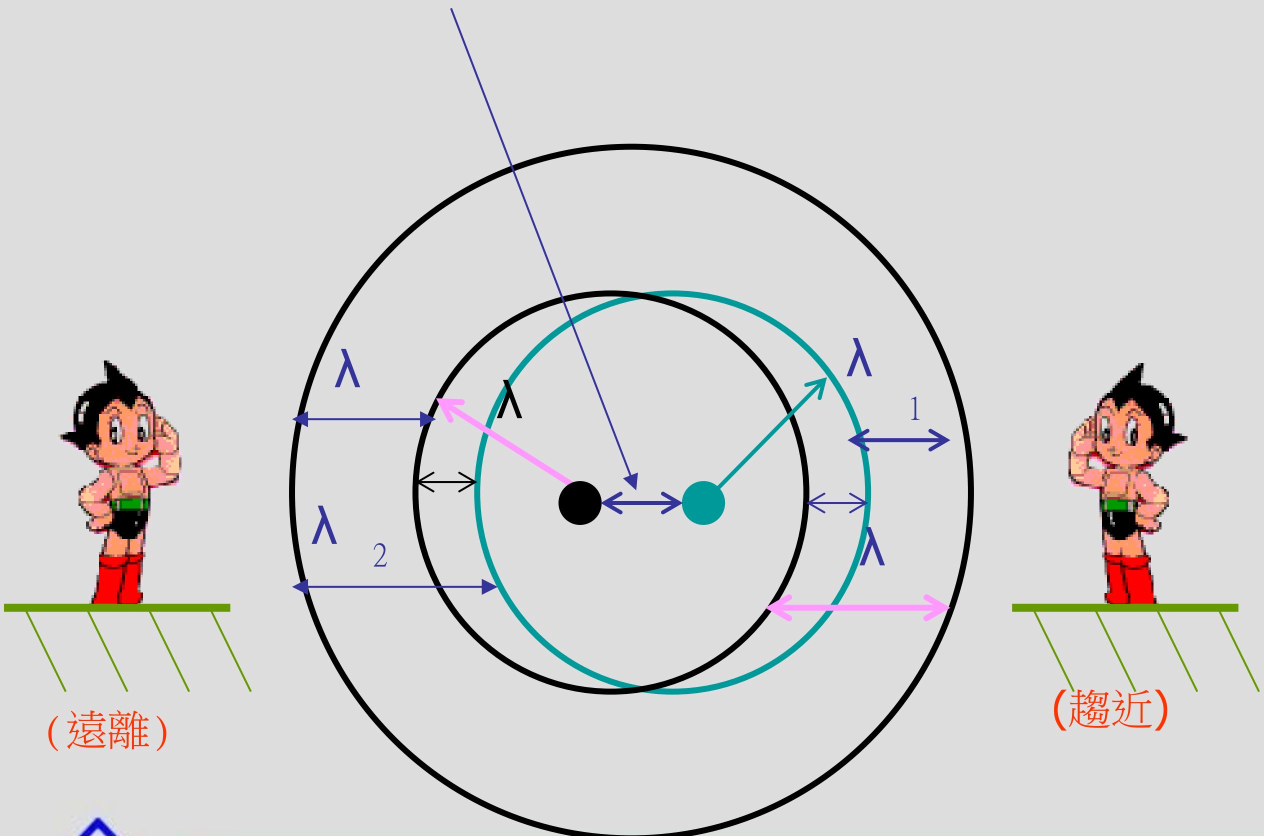
$$T \cdot V_s = \leftrightarrow = \Delta S$$

$$\lambda = \lambda_1 + \Delta S$$

$$\lambda = \lambda_2 - \Delta S$$

$$\lambda_1 = \lambda - \frac{V_s}{f_s}$$

$$\lambda_2 = \lambda + \leftrightarrow = \lambda + \frac{V_s}{f_s}$$



$$S \rightarrow S' \left(t = T = \frac{1}{f_s}\right)$$

$$\Delta S = V_s T = \frac{V_s}{f_s}$$

$$\lambda_1 = \lambda - \Delta S = \lambda - \frac{V_s}{f_s}$$ （趨近） $\lambda_1 < \lambda$

$$\lambda_2 = \lambda + \Delta S = \lambda + \frac{V_s}{f_s}$$ （遠離） $\lambda_2 > \lambda$

V波源波速不變 $\Rightarrow f_{0(趨近)} = \frac{V}{\lambda_1} > f_s = \frac{V}{\lambda}$

$$f_{0(遠離)} = \frac{V}{\lambda_2} < f_s = \frac{V}{\lambda}$$

$\Rightarrow f_0 = \dfrac{V}{\lambda_1}$ (趨近)

$$= \frac{V}{\lambda - V_s/f_s}$$

$$= \frac{V}{V/f_s - V_s/f_s}$$

$$= \frac{f_s}{1 - V_s/V}$$ (頻率升高)

$\Rightarrow f_0 = \dfrac{V}{\lambda_2}$ (遠離)

$$= f_s \times \frac{V}{V + V_s}$$ (頻率降低)

13. EM Wave's Doppler Effect

EM: Electro Magnetic→電磁

$$f_0 - f_s = \Delta f = \pm f_s \frac{\Delta V}{C}$$

+趨近

-遠離

和 $$f_0 = f_s\left(1 \pm \frac{\Delta V}{V}\right)$$ 是相同

14、延遲波的表示式

若延遲 X 亦即延遲 $t_0 = \frac{x}{v}$

(Delay)

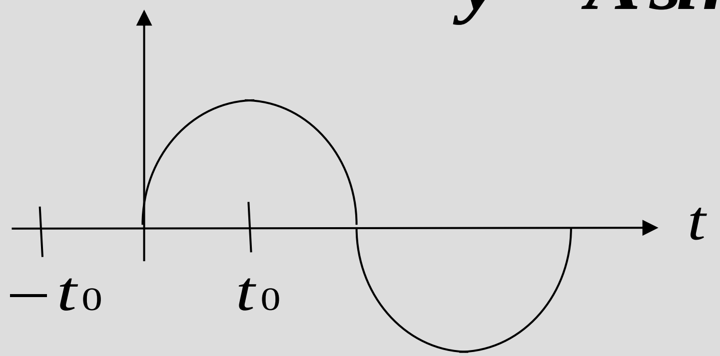
$$y = A\sin\omega t' \quad [\,t' = t - t_0\,]$$

$$= A\sin\omega(t - \frac{x}{v}) \quad (\text{延遲波})$$

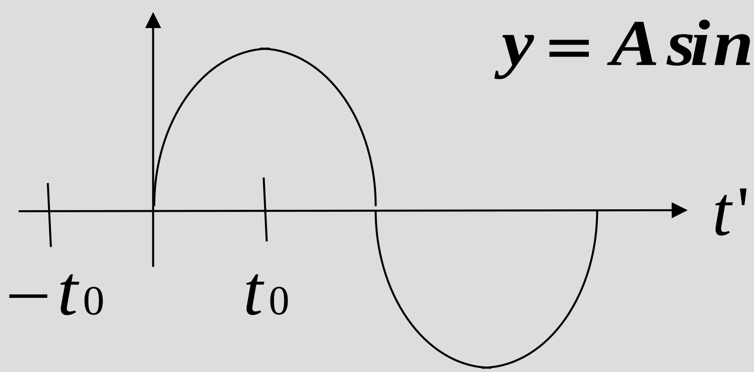
$$y = A\sin\omega t''[t'' = t + t_0]$$

$$= A\sin(t + \frac{x}{v})(\text{提前波})$$

$$y = A\sin\omega t$$



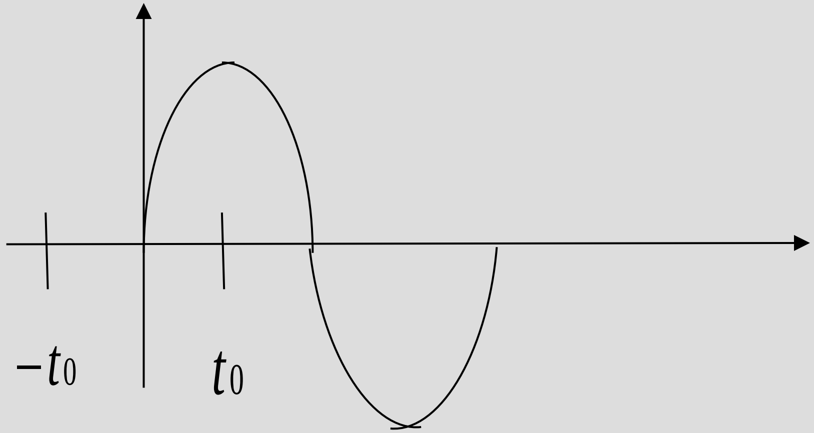
$$y = A\sin\omega t'$$



（延遲波）

$$t - t_0 = t'$$

$$y = A\sin\omega t''$$



$t + t_0 = t''$

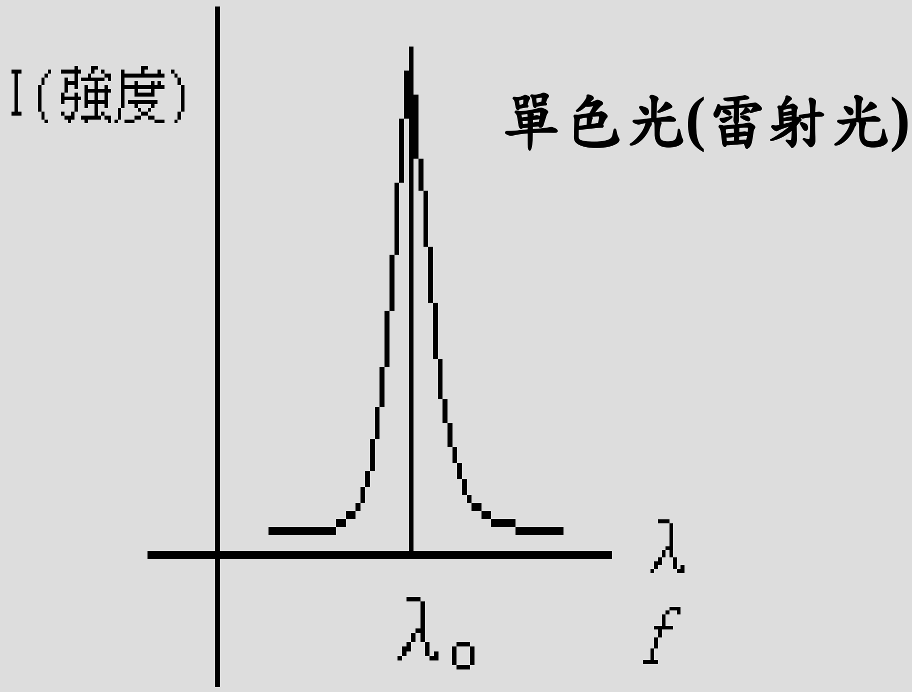
（提前波）

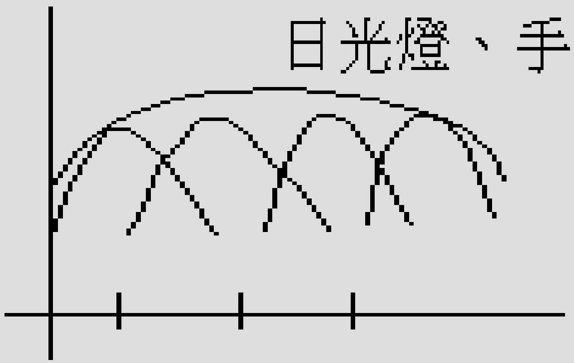
$$y = A\sin\omega t'' \; [t'' = t + t_0]$$

$$= A\sin\omega\left(t + \frac{x}{V}\right)(\text{提前})$$

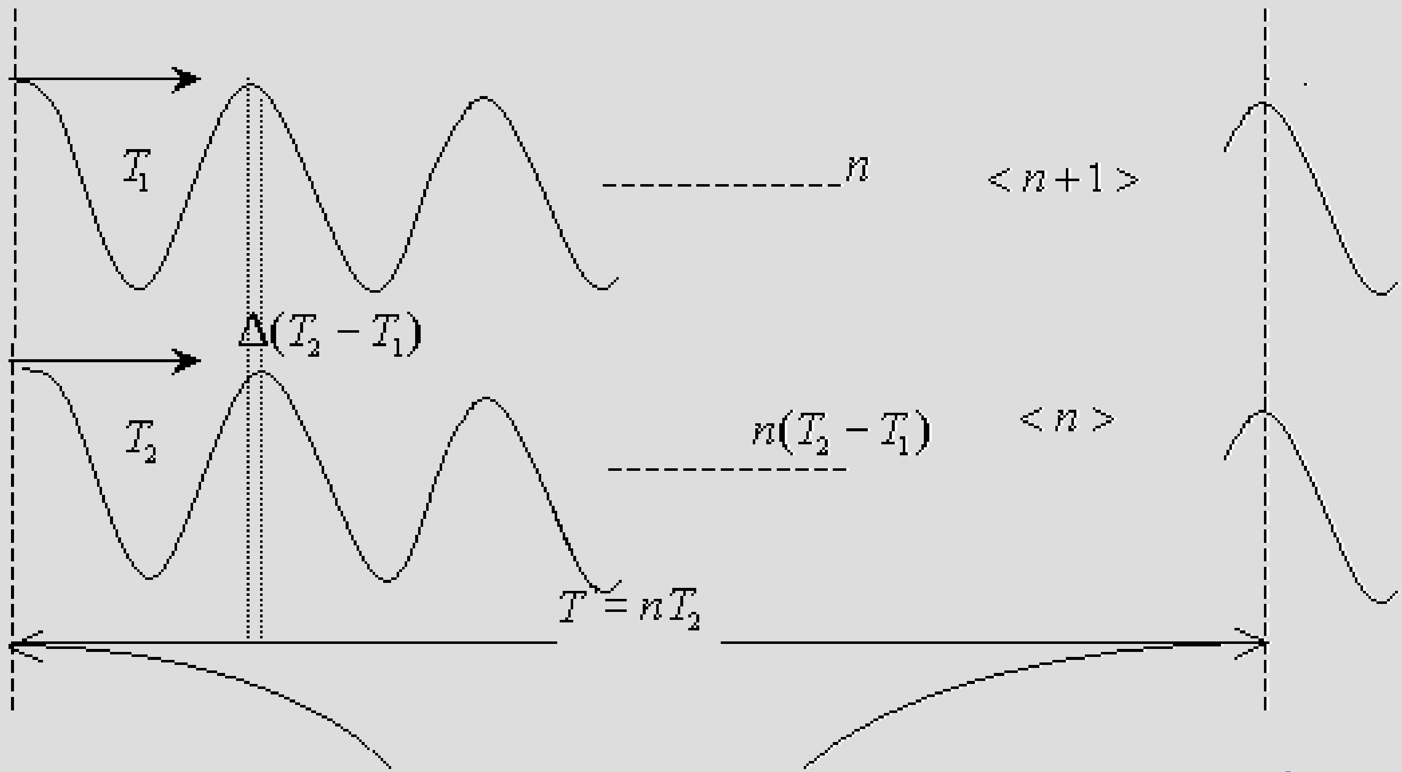
$(\text{若波向} -x \text{方向移動} V \rightarrow -V)$

15. 兩波源有固定之相位差(包括同相)關係謂之“共相性(Coherent)”。





16、BEAT(節拍)—Interference of two waves ($f1 \approx f2$)



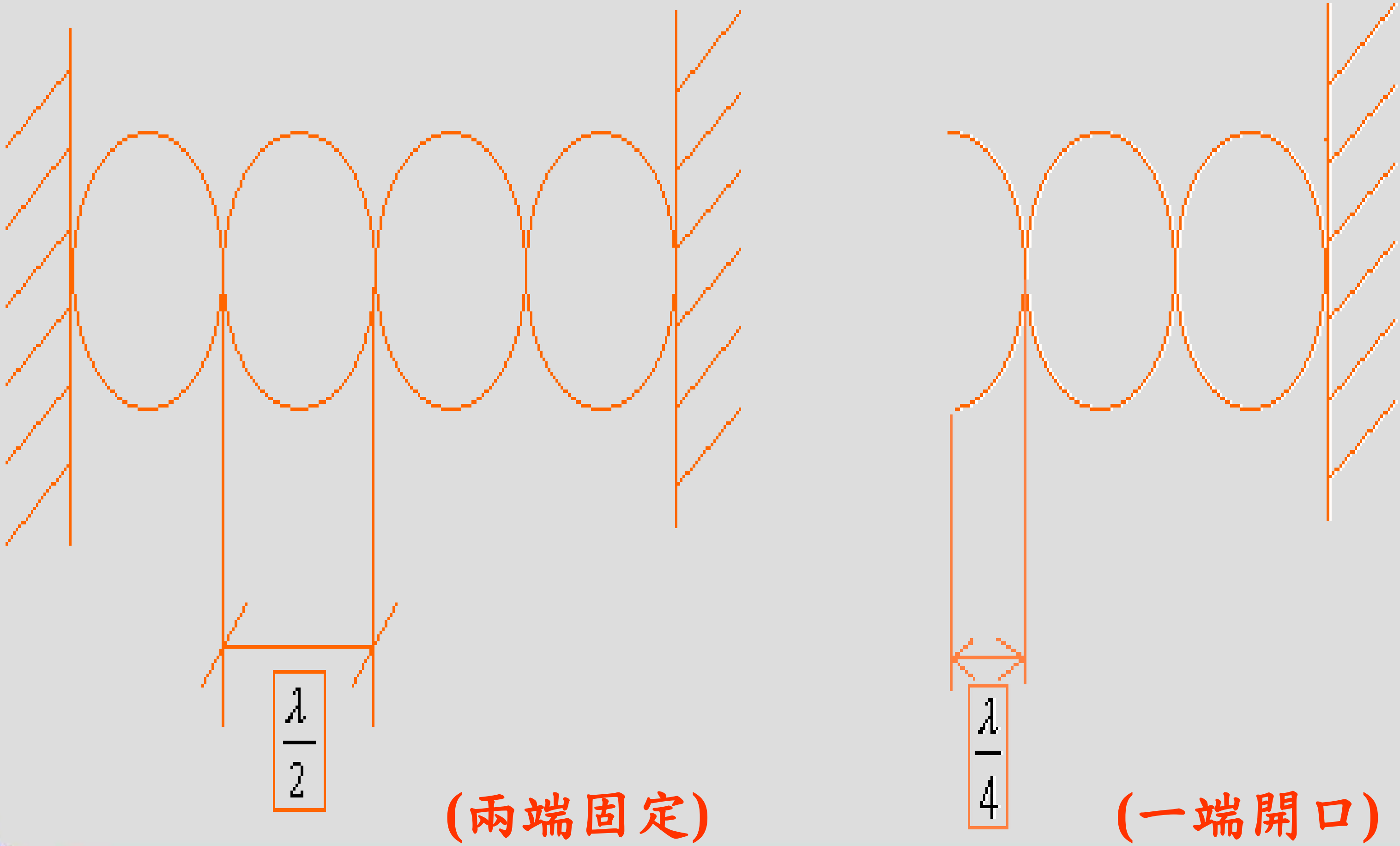
同相

$$n(T_2 - T_1) = T_1 \qquad n = \frac{T_1}{(T_2 - T_1)}$$

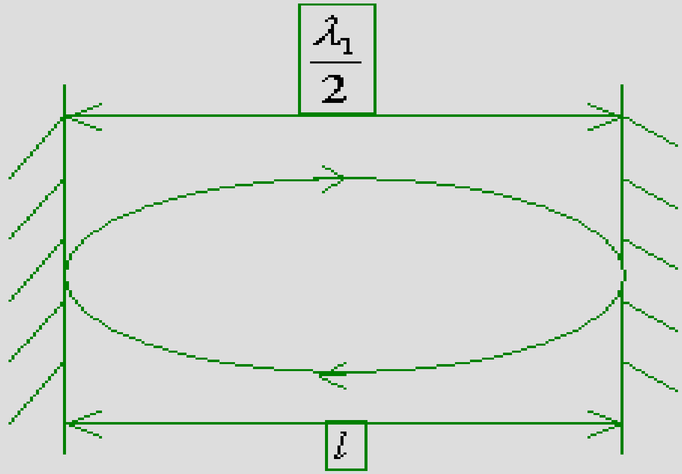
$$T = nT_2 = \frac{T_2 T_1}{T_2 - T_1}$$

$$\frac{1}{f_{BEAT}} = \frac{T_2 T_1}{T_2 - T_1} \Rightarrow f_{BEAT} = \frac{T_2 - T_1}{T_2 T_1} = \frac{1}{T_1} - \frac{1}{T_2} = f_1 - f_2$$

17、Standing Wave 駐波



Fundamental or First Harmonic

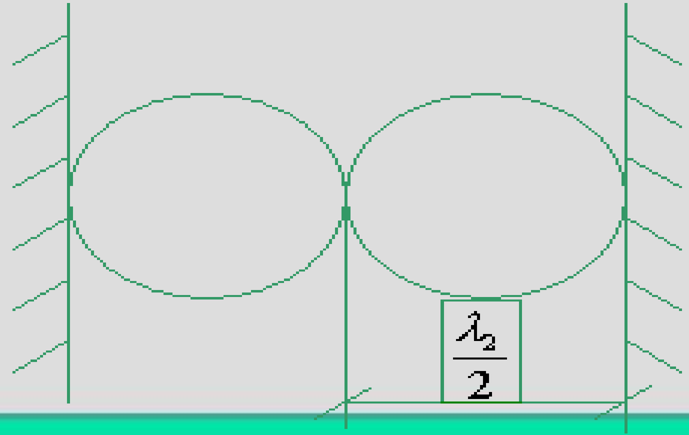


$$f_1 \Rightarrow l = \frac{\lambda_1}{2}, \lambda_1 = 2l$$

$$f_1 = \frac{V}{\lambda_1} = \frac{V}{2l}$$

$$= \frac{\sqrt{\frac{F}{\mu}}}{2l}$$

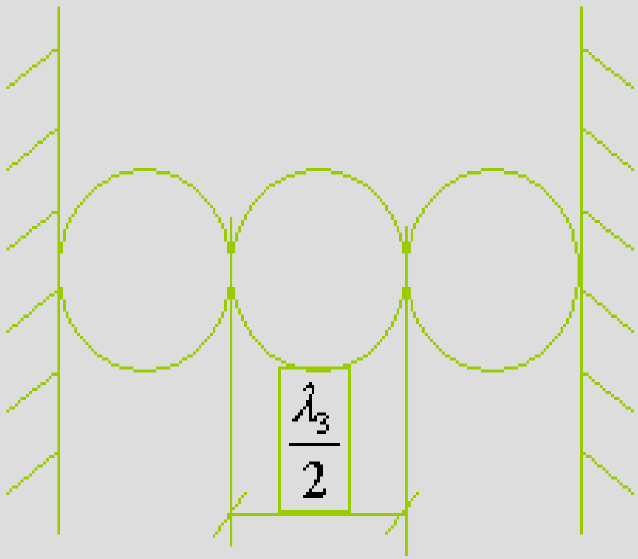
Second Harmonic



$$f_2 \Rightarrow l = \frac{2\lambda_2}{2}, \lambda_2 = l$$

$$f_2 = \frac{V}{\lambda_2} = \frac{V}{l}$$

Third Harmonic



$$f_3 \Rightarrow l = \frac{3\lambda_3}{2}, \lambda_3 = \frac{2}{3}l$$

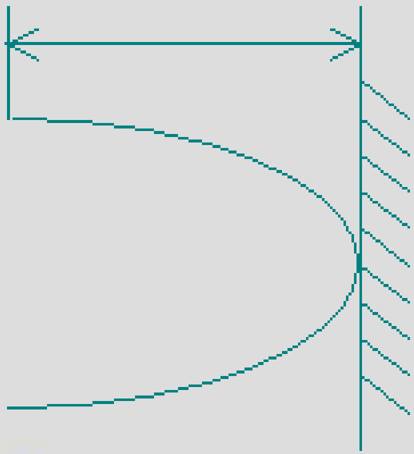
$$f_3 = \frac{V}{\lambda_3} = \frac{3V}{2l}$$

Nth Harmonic $f_n = \frac{V}{\lambda_n} = \frac{nV}{2l}$

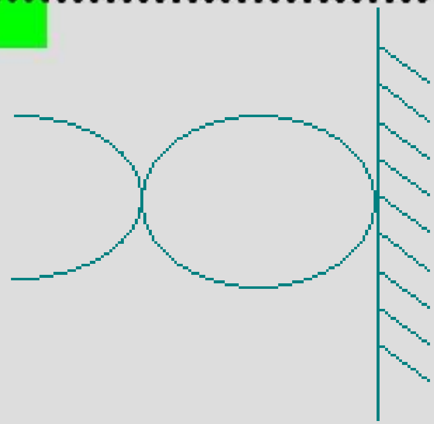
【兩端固定和兩端開口】的駐波是以 $\frac{\lambda}{2}$(1波包)為基本數

18、（繩波與兩端開<閉>口的音波）駐波公式是一致的，而（一端開口另一端封閉音波）的駐波公式稍有不同。

($\frac{\lambda}{4}+(n-1)$ 個波包之基本數） ➔ 為基本數 $l=\frac{\lambda_{(2n-1)}}{4}+(n-1)\frac{\lambda_{(2n-1)}}{2}$

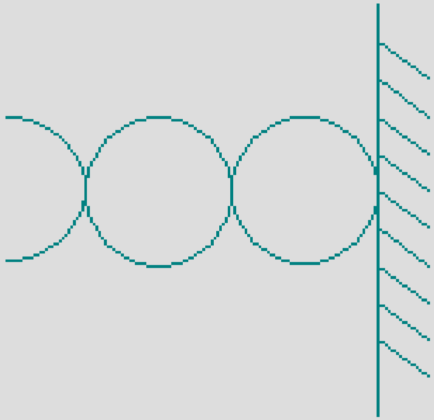


$l=\frac{\lambda_1}{4}\Rightarrow f_1=\frac{}{\lambda_1}=\frac{}{4l}$ (1st Harmonic)



$$l = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} \Rightarrow f_3 = \frac{3V}{4l} = 3f_1$$

$$n = 2$$



$$l = \frac{\lambda_5}{4} + \lambda_5 \Rightarrow f_5 = \frac{5V}{4l} = 5f_1$$

$$n = 3$$

$$l = \frac{\lambda_{(2n-1)}}{4} + \frac{\lambda_{(2n-1)}}{2}(n-1) = \frac{(2n-1)}{4}\lambda_{(2n-1)}$$

$$\Rightarrow f_{(2n-1)} = \frac{V}{\lambda_{(2n-1)}} = \frac{(2n-1)V}{4l}$$

$$(n = 1, 2, 3, ...)$$