Chapter 7. 靜電學 Statics Electricity

1.靜電學：靜止的電荷相關物理量的分析

電荷→電量

電場→ $\vec{E}_{主}$ 電力(庫倫力) $\vec{F}_{客} = q_{客}\vec{E}_{主}$

電位→ $V_{主}$ 電位能 $U_{客} = q_{客}V_{主}$

電位能 $U_E = -\int \vec{F}_E \cdot d\vec{r}$

重力位能 $U_G = -\int \vec{F}_G \cdot d\vec{r}$

$$V_{主} = \frac{U_{客}}{q_{客}} = \frac{-\int \vec{F}_{客} \cdot d\vec{r}}{q_{客}} = -\int \frac{\vec{F}_{客}}{q_{客}} \cdot dr = -\int \vec{E}_{主} \cdot dr$$

→應用元件→電容

※任何帶電物質電量應為電子量的整數倍(原子核外)

◎◎ 靜電學

1. 電荷 [±]

2. 電量 [C]

3. 電場

$$\vec{E}_{主} = \frac{kQ_{主}}{r^2} \quad (Q_{主})$$

電位

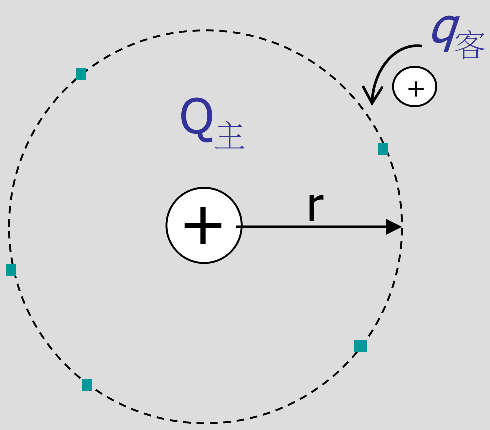
$$V_{主}(r) = \frac{U_{客}(r)}{q_{客}} = -\int \vec{E}_{主} \cdot d\vec{r} = \frac{kQ_{主}}{r} \quad (Q_{主})$$

4. 電力

$$\vec{F}_{客} = q_{客}\vec{E}_{主} = \frac{kQ_{主}q_{客}}{r^2} \quad (q_{客})$$

5. 電位能

$$U_{客}(r) = -\int \vec{F}_{客} \cdot d\vec{r} = \frac{kQ_{主}q}{r} \quad (q_{客})$$



$$\vec{F}_{客} = q_{客}\vec{E}_{主} = \frac{kQ_{主}q_{客}}{r^2}$$

$$U_{客} = q_{客}V_{主} = \frac{kQ_{主}q_{客}}{r}$$

2. 電荷 (Electric Charge)

→ 目前基本電荷量 (原子核除外) → 電子

= 電子電量 $e = -1.6 \times 10^{-19} C$

正電荷 (Positive Charge)

負電荷 (Negative Charge) 電子是負電荷

任何帶電或有電荷分佈之物體所帶電量【$Q = \pm Ne$】

EXAMPLE 21.2 FORCE BETWEEN TWO POINT CHARGES

Two point charges, $q_1 = +25$ nC and $q_2 = -75$ nC, are separated by a distance $r = 3.0$ cm (**Fig. 21.12a**). Find the magnitude and direction of the electric force (a) that q_1 exerts on q_2 and (b) that q_2 exerts on q_1.

SOLUTION

IDENTIFY and SET UP: This problem asks for the electric forces that two charges exert on each other. We use Coulomb's law, Eq. (21.2), to calculate the magnitudes of the forces. The signs of the charges will determine the directions of the forces.

EXECUTE: (a) After converting the units of r to meters and the units of q_1 and q_2 to coulombs, Eq. (21.2) gives us

$$F_{1 \text{ on } 2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

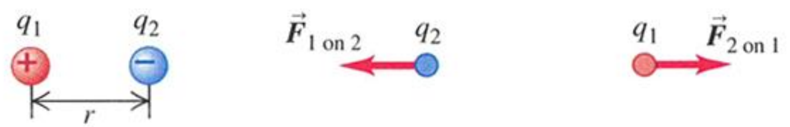
$$= (9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{|(+25 \times 10^{-9}\ \text{C})(-75 \times 10^{-9}\ \text{C})|}{(0.030\ \text{m})^2}$$

$$= 0.019\ \text{N}$$

The charges have opposite signs, so the force is attractive (to the left in Fig. 21.12b); that is, the force that acts on q_2 is directed toward q_1 along the line joining the two charges.

21.12 What force does q_1 exert on q_2, and what force does q_2 exert on q_1? Gravitational forces are negligible.

(a) The two charges (b) Free-body diagram for charge q_2 (c) Free-body diagram for charge q_1



(b) Proceeding as in part (a), we have

$$F_{2 \text{ on } 1} = \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_1|}{r^2} = F_{1 \text{ on } 2} = 0.019\ \text{N}$$

The attractive force that acts on q_1 is to the right, toward q_2 (Fig. 21.12c).

EVALUATE: Newton's third law applies to the electric force. Even though the charges have different magnitudes, the magnitude of the force that q_2 exerts on q_1 is the same as the magnitude of the force that q_1 exerts on q_2, and these two forces are in opposite directions.

SOLUTION

q_2電力 $\vec{F}_{客}$ ➔ q_1電場 $\vec{E}_{主}=\dfrac{\vec{F}_{客}}{q_{客}}=\dfrac{kQ_{主}}{r^2}$

➔ q_2電位能 $U_{客}(r)=-\int \vec{F}_{客}\cdot d\vec{r}=\dfrac{kQ_{主}q}{r}$

➔ q_1電位 $V_{主}(r)=\dfrac{U_{客}(r)}{q_{客}}=\dfrac{kQ_{主}}{r}$

EXAMPLE 21.4 VECTOR ADDITION OF ELECTRIC FORCES IN A PLANE

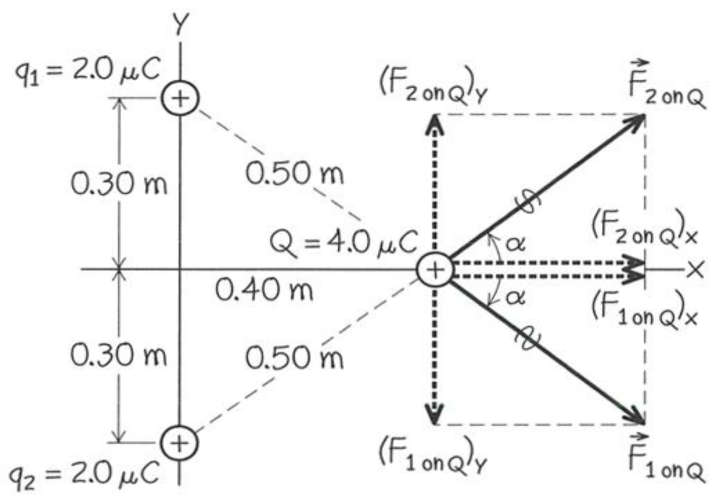
SOLUTION

Two equal positive charges $q_1 = q_2 = 2.0\ \mu C$ are located at $x = 0, y = 0.30$ m and $x = 0, y = -0.30$ m, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu C$ at $x = 0.40$ m, $y = 0$?

SOLUTION

IDENTIFY and SET UP: As in Example 21.3, we must compute the force that each charge exerts on Q and then find the vector sum of those forces. **Figure 21.14** shows the situation. Since the three charges do not all lie on a line, the best way to calculate the forces is to use components.

21.14 Our sketch for this problem.



EXECUTE: Figure 21.14 shows the forces $\vec{F}_{1\text{ on }Q}$ and $\vec{F}_{2\text{ on }Q}$ due to the identical charges q_1 and q_2, which are at equal distances from Q. From Coulomb's law, *both* forces have magnitude

$$F_{1\text{ or }2\text{ on }Q} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(4.0 \times 10^{-6}\ \text{C})(2.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2} = 0.29\ \text{N}$$

The x-components of the two forces are equal:

$$(F_{1\text{ or }2\text{ on }Q})_x = (F_{1\text{ or }2\text{ on }Q})\cos\alpha = (0.29\ \text{N})\frac{0.40\ \text{m}}{0.50\ \text{m}} = 0.23\ \text{N}$$

From symmetry we see that the y-components of the two forces are equal and opposite. Hence their sum is zero and the total force $\vec{F}$ on Q has only an x-component $F_x = 0.23\ \text{N} + 0.23\ \text{N} = 0.46\ \text{N}$. The total force on Q is in the $+x$-direction, with magnitude 0.46 N.

EVALUATE: The total force on Q points neither directly away from q_1 nor directly away from q_2. Rather, this direction is a compromise that points away from the *system* of charges q_1 and q_2. Can you see that the total force would *not* be in the $+x$-direction if q_1 and q_2 were not equal or if the geometrical arrangement of the changes were not so symmetric?

3. 電場 (Electric Field)

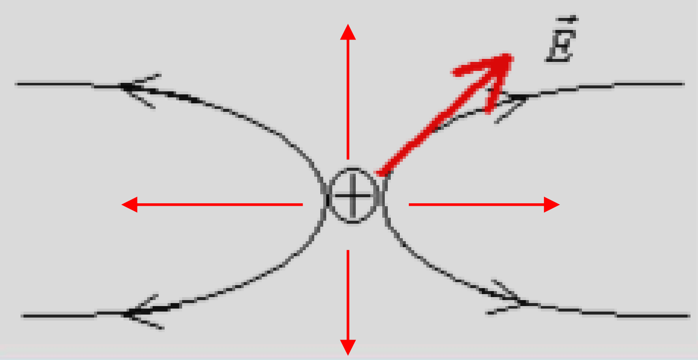
※電場定義：$\vec{E}_{主} = \dfrac{\vec{F}_{客}}{q_{客}} = \dfrac{kQ_{主}}{r^2}\dfrac{\vec{r}}{|\vec{r}|}$

→ $Q_{主}$ 所輻射的電力線所形成的電場【$\vec{E}_{主}$】

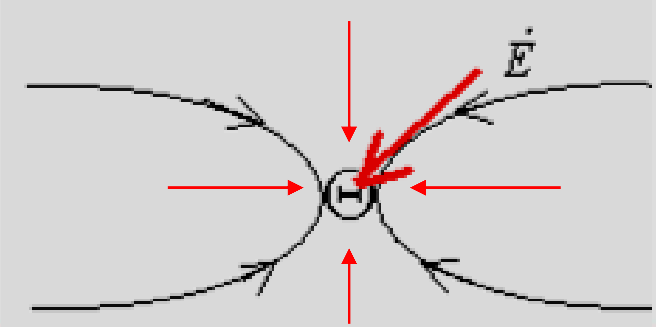
$Q_{主}$之電力線切線方向即電場 $\vec{E}_{主}$之方向

→ 電荷之電場方向 $E_{主} = \dfrac{kQ_{主}}{r^2}$ $\quad \vec{E}_{主} = \dfrac{kQ_{主}}{r^2}\dfrac{\vec{r}}{|\vec{r}|} = \dfrac{kQ_{主}}{r^3}\vec{r}$

正電荷電場輻射向外



負電荷電場輻射向內



EXAMPLE 21.6 ELECTRIC-FIELD VECTOR FOR A POINT CHARGE

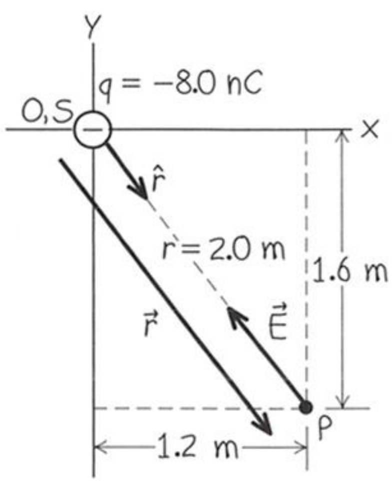
SOLUTION

A point charge $q = -8.0\text{ nC}$ is located at the origin. Find the electric-field vector at the field point $x = 1.2\text{ m}$, $y = -1.6\text{ m}$.

SOLUTION

IDENTIFY and SET UP: We must find the electric-field *vector* $\vec{E}$ due to a point charge. **Figure 21.19** shows the situation. We use Eq. (21.7); to do this, we must find the distance r from the source point S (the position of the charge q, which in this example is at the origin O) to the field point P, and we must obtain an expression for the unit vector $\hat{r} = \vec{r}/r$ that points from S to P.

21.19 Our sketch for this problem.



EXECUTE: The distance from S to P is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\text{ m})^2 + (-1.6\text{ m})^2} = 2.0\text{ m}$$

The unit vector $\hat{r}$ is then

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{r}$$

$$= \frac{(1.2\text{ m})\hat{\imath} + (-1.6\text{ m})\hat{\jmath}}{2.0\text{ m}} = 0.60\hat{\imath} - 0.80\hat{\jmath}$$

Then, from Eq. (21.7),

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$= (9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-8.0 \times 10^{-9}\text{ C})}{(2.0\text{ m})^2}(0.60\hat{\imath} - 0.80\hat{\jmath})$$

$$= (-11\text{ N/C})\hat{\imath} + (14\text{ N/C})\hat{\jmath}$$

EVALUATE: Since q is negative, $\vec{E}$ points from the field point to the charge (the source point), in the direction opposite to $\hat{r}$ (compare Fig. 21.17c). We leave the calculation of the magnitude and direction of $\vec{E}$ to you (see Exercise 21.30).

EXAMPLE 21.8 FIELD OF AN ELECTRIC DIPOLE

SOLUTION

Point charges $q_1 = +12$ nC and $q_2 = -12$ nC are 0.100 m apart (**Fig. 21.22**). (Such pairs of point charges with equal magnitude and opposite sign are called *electric dipoles*.) Compute the electric field caused by q_1, the field caused by q_2, and the total field (a) at point a; (b) at point b; and (c) at point c.

SOLUTION

IDENTIFY and SET UP: We must find the total electric field at various points due to two point charges. We use the principle of superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2$. Figure 21.22 shows the coordinate system and the locations of the field points a, b, and c.

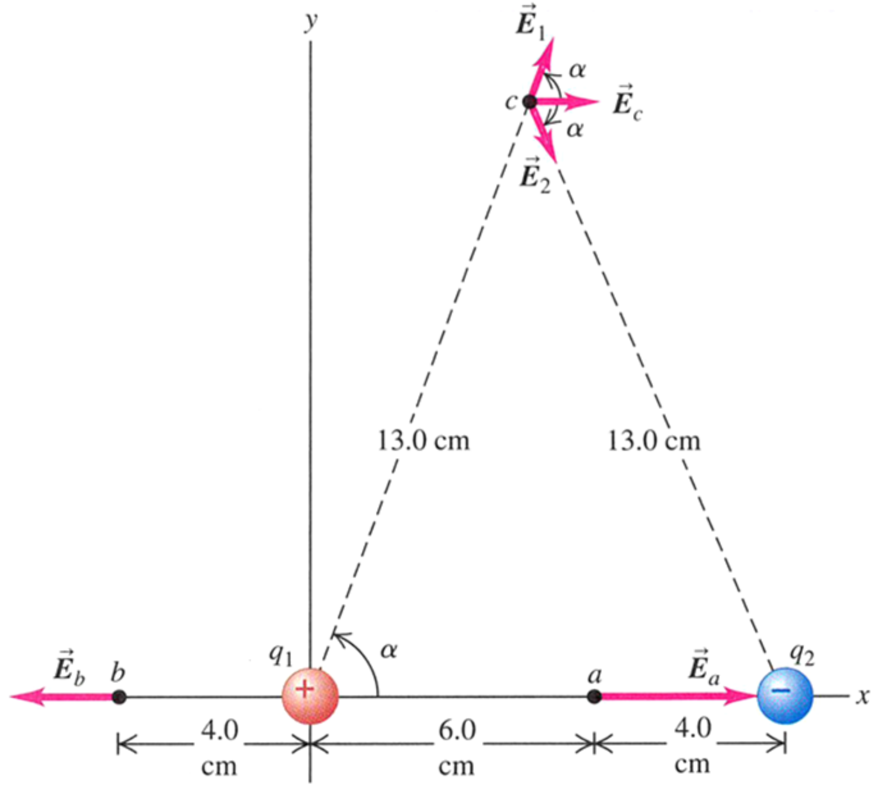
EXECUTE: At each field point, $\vec{E}$ depends on $\vec{E}_1$ and $\vec{E}_2$ there; we first calculate the magnitudes E_1 and E_2 at each field point. At a the magnitude of the field $\vec{E}_{1a}$ caused by q_1 is

$$E_{1a} = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2}$$

$$= (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{12 \times 10^{-9}\ \mathrm{C}}{(0.060\ \mathrm{m})^2}$$

$$= 3.0 \times 10^4\ \mathrm{N/C}$$

21.22 Electric field at three points, a, b, and c, set up by charges q_1 and q_2, which form an electric dipole.



We calculate the other field magnitudes in a similar way. The results are

$$E_{1a} = 3.0 \times 10^4\ \text{N/C}$$

$$E_{1b} = 6.8 \times 10^4\ \text{N/C}$$

$$E_{1c} = 6.39 \times 10^3\ \text{N/C}$$

$$E_{2a} = 6.8 \times 10^4\ \text{N/C}$$

$$E_{2b} = 0.55 \times 10^4\ \text{N/C}$$

$$E_{2c} = E_{1c} = 6.39 \times 10^3\ \text{N/C}$$

The *directions* of the corresponding fields are in all cases *away* from the positive charge q_1 and *toward* the negative charge q_2.

(a) At a, $\vec{E}_{1a}$ and $\vec{E}_{2a}$ are both directed to the right, so

$$\vec{E}_a = E_{1a}\hat{\imath} + E_{2a}\hat{\imath} = (9.8 \times 10^4\ \text{N/C})\hat{\imath}$$

(b) At b, $\vec{E}_{1b}$ is directed to the left and $\vec{E}_{2b}$ is directed to the right, so

$$\vec{E}_b = -E_{1b}\hat{\imath} + E_{2b}\hat{\imath} = (-6.2 \times 10^4\ \text{N/C})\hat{\imath}$$

(c) Figure 21.22 shows the directions of $\vec{E}_1$ and $\vec{E}_2$ at c. Both vectors have the same x-component:

$$E_{1cx} = E_{2cx} = E_{1c}\cos\alpha = (6.39 \times 10^3\ \text{N/C})\left(\tfrac{5}{13}\right)$$
$$= 2.46 \times 10^3\ \text{N/C}$$

From symmetry, E_{1y} and E_{2y} are equal and opposite, so their sum is zero. Hence

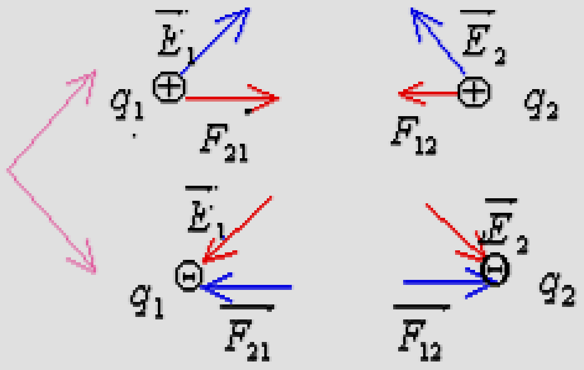
$$\vec{E}_c = 2(2.46 \times 10^3\ \text{N/C})\hat{\imath} = (4.9 \times 10^3\ \text{N/C})\hat{\imath}$$

EVALUATE: We can also find $\vec{E}_c$ by using Eq. (21.7) for the field of a point charge. The displacement vector $\vec{r}_1$ from q_1 to point c is $\vec{r}_1 = r\cos\alpha\hat{\imath} + r\sin\alpha\hat{\jmath}$. Hence the unit vector that points from q_1 to point c is $\hat{r}_1 = \vec{r}_1/r = \cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$. By symmetry, the unit vector that points from q_2 to point c has the opposite x-component but the same y-component: $\hat{r}_2 = -\cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$. We can now use Eq. (21.7) to write the fields $\vec{E}_{1c}$ and $\vec{E}_{2c}$ at c in vector form, then find their sum. Since $q_2 = -q_1$ and the distance r to c is the same for both charges,

$$\vec{E}_c = \vec{E}_{1c} + \vec{E}_{2c} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}_1 + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{r}_2$$
$$= \frac{1}{4\pi\epsilon_0 r^2}(q_1\hat{r}_1 + q_2\hat{r}_2)$$
$$= \frac{q_1}{4\pi\epsilon_0 r^2}(\hat{r}_1 - \hat{r}_2)$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(2\cos\alpha\hat{\imath})$$
$$= 2(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.13\ \text{m})^2}\left(\tfrac{5}{13}\right)\hat{\imath}$$
$$= (4.9 \times 10^3\ \text{N/C})\hat{\imath}$$

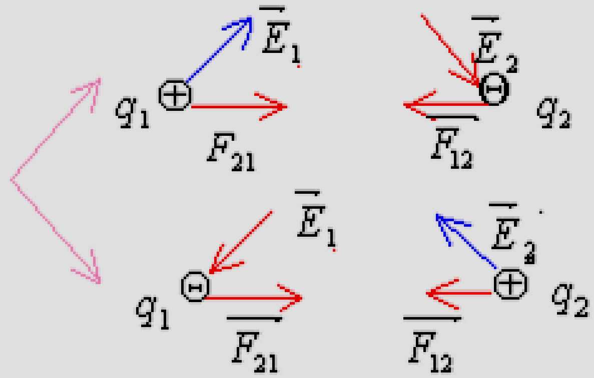
This is the same as we calculated in part (c).

☆ 同性相斥



$$\vec{E}_1 \Rightarrow \vec{F}_{12} = q_2\vec{E}_1$$
$$\vec{E}_2 \Rightarrow \vec{F}_{21} = q_1\vec{E}_1$$
$$\vec{E}_1 \Rightarrow \vec{F}_{12} = q_2\vec{E}_1$$
$$\vec{E}_2 \Rightarrow \vec{F}_{21} = q_1\vec{E}_2$$

☆ 異性相吸



$$\vec{E}_1 \Rightarrow \vec{F}_{12} = q_2\vec{E}_1$$
$$\vec{E}_2 \Rightarrow \vec{F}_{21} = q_1\vec{E}_1$$
$$\vec{E}_1 \Rightarrow \vec{F}_{12} = q_2\vec{E}_1$$
$$\vec{E}_2 \Rightarrow \vec{F}_{21} = q_1\vec{E}_2$$

會提這個問題，可見得你對物理有相當不錯的了解。

當初庫倫提出其所觀察到的現象(1781 年)，兩帶電靜電體間會出現相吸或相斥的作用，讓懸掛的細絲出現旋轉的扭力矩現象。庫倫試著用流體的概念解釋他觀察到的現象。

後人發現，電荷間的作用力與兩電荷的電量乘積成正比，和電荷間的距離平方成反比。其中的比率常數即為庫倫常數。

$$F = k_e \frac{q_1 q_2}{r^2}$$

到了 1835 年高斯利用流體流線與通量的觀念(還是類似庫倫的想法)，說明介質中電通量和電荷的關係。

1845 年法拉第首先用場(Field)的字來描述電磁感應的現象，接著在 1861 年馬克斯威爾將高斯定律、安培定律、和法拉第定律分別寫出常見的微分表示式，並由此微分的高斯定律可直接導出庫倫定率，庫倫定律等於

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}.$$

庫倫常數則為

$$k_e = \frac{1}{4\pi\varepsilon_0} = \frac{c^2\ \mu_0}{4\pi} = c^2 \cdot 10^{-7}\ \mathrm{H}\cdot\mathrm{m}^{-1}$$
$$= 8.987\ 551\ 787 \times 10^9\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2.$$

式中 $\varepsilon_0 = 1/(\mu_0 c^2) \approx 8.854187817\times10^{-12}\ \mathrm{F}\cdot\mathrm{m}^{-1}$ 為介質(真空)的電導係數或介電係數。

庫倫常數所定義的庫倫力是一種超距力，無法敘述此力如何傳遞，也無法描述介質對電場的影響。(介質具有電偶極，電偶極會被外加電場極化，也會建立介質的電場)。

當我們引進介質的介電係數後，馬克斯威爾方程式即可得出電磁波的運動方程式，定得出電磁波的傳播速度即光速。光速為

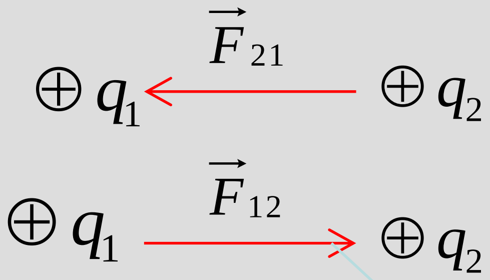
$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}.$$

介質的折射率為 $n = \sqrt{\epsilon_r \mu_r},$ 。可知當我們引介電係數進庫倫常數中，可得場的概念。電荷會建立電場然後電場再和其他電荷產生力的作用。介質也會影響電場，電場隨時間變化時會形成電磁波，介質的介電係數可決定介質的折射率。

4. 庫倫力 (Coulomb Force)

(1)電場 $\vec{E}_{主}$ 施予 $q_{客}$ 的作用力

(2)兩個電荷之間的作用力



$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = \vec{F}_{客}\ (庫倫力) = k\frac{Q_{主}q_{客}}{r^2} = k\frac{q_1 q_2}{r^2}$$

$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = \left|\vec{F}_{客}\right|$ 大小相等/方向相反(作用力和反作用力)

Maxwell的方程式偏微分形式

$\nabla \cdot D = \rho$ ....(1)　靜電學定律/靜電荷產生電通密度

$\nabla \cdot B = 0$ ……………....(2)　靜磁學定律/靜磁場產生磁通迴路

時變電磁場

$\nabla \times E = -\partial B/\partial t$ ....(3)　(法拉第定律/磁生電)

$\nabla \times H = J + \partial D/\partial t$ .....(4)　(安培定律/電生磁)

其中的　E：電場強度　D：電通密度，又稱電位移向量
　　　　B：磁通密度　H：磁場強度
　　　　J：電流面密度　ρ：體電荷密度(純量)

靜電學定律積分形式和偏微分形式互等

積分式 $\Phi = \oiint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o}$

一度空間 $E_x \rightarrow$ yz 面

$A = dydz$

$\int E_x \cdot dydz = \frac{Q}{\varepsilon_o}$

$E_x = \frac{Q}{\varepsilon_o dydz}$

兩者相等

微分式 $\nabla \cdot \vec{D} = \rho$

$(\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z})$

$\vec{D} = \varepsilon_o \overrightarrow{E_x}$

$\frac{\partial}{\cancel{\partial x}}(\varepsilon_o E_x) = \frac{Q}{V} = \frac{Q}{\cancel{dx}(dydz)}$

$\varepsilon_o E_x = \frac{Q}{dydz}$

$E_x = \frac{Q}{\varepsilon_o dydz}$

用史托克定理〈Stoke's Theorem〉將Maxwell方程式法拉第定律的

偏微分形式　　轉成　　積分型態

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

⟺

$$\oint_C \vec{E} \cdot d\vec{l} = \xi mf$$

$$= -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{S} = -\frac{d\Phi_S}{dt}$$

法拉第定律積分形式和偏微分形式互等

法拉第定律積分形式和偏微分形式互等

從微分式出發 $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

同時對曲面S積分 $$\int_S (\nabla \times \vec{E}) \cdot d\vec{A} = -\int_S \frac{\partial}{\partial t} \vec{B} \cdot d\vec{A}$$

根據 Stokes' Theorem $$\int_S (\nabla \times \vec{E}) \cdot d\vec{A} = \oint_P \vec{E} \cdot d\vec{l}$$

所以 $$\oint_P \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial}{\partial t} \vec{B} \cdot d\vec{A}$$

$$-\int_S \frac{\partial}{\partial t} \vec{B} \cdot d\vec{A} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} = -\frac{d}{dt} \Phi_{B,S}$$

法拉第定律積分形式和偏微分形式互等

$$\Phi_B = \oiint \vec{B}\cdot d\vec{A}$$

$$\Rightarrow \varepsilon = \oint \overrightarrow{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d}{dt}\vec{B}_z \cdot d\vec{A}$$

$$= -\frac{d}{dt}\vec{B}_z \cdot (d\vec{x}d\vec{y})$$

$$= -\frac{d}{dt}\vec{B}_z \cdot d\vec{x}d\vec{y}$$

(設B方向為z軸 $\therefore d\vec{A} = dxdy$)

$$\nabla\times\vec{E} = \frac{\partial\vec{E}y}{\partial x} - \frac{\partial\vec{E}x}{\partial y} = -\frac{d\vec{B}_z}{dt}$$

$$\Rightarrow \nabla\times\vec{E}\cdot d\vec{A}$$

$$= \left(\frac{\partial\vec{E}_y}{\partial x} - \frac{\partial\vec{E}_x}{\partial y}\right)\cdot dxdy$$

$$= \partial\vec{E}_y \cdot d\vec{y} - \partial\vec{E}_x \cdot d\vec{x}$$

$= \oint \overrightarrow{E}\cdot d\vec{l}$ (同軸內積 $\vec{E}_x \cdot d\vec{x}$ 和 $\vec{E}_y \cdot d\vec{y}$)

$$= -\frac{d\vec{B}_z}{dt}\cdot d\vec{x}d\vec{y}$$

5. 電位 (Electric Potential)/電位能(Electric Potential Energy)

電位定義 $V_{主} = \frac{U_{客}}{q_{客}} = \frac{kQ_{主}}{r}$

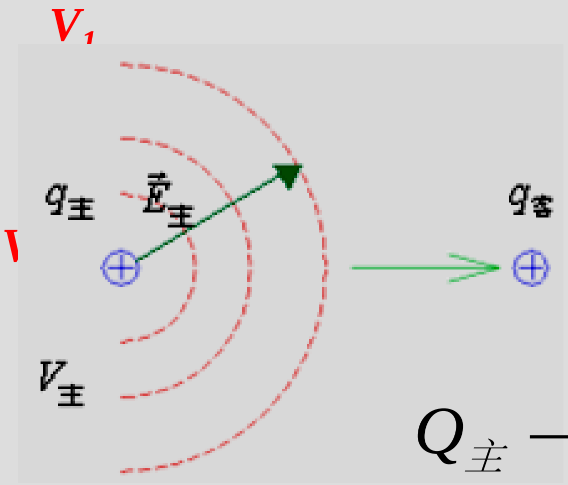
→ 庫倫力(電力)和萬有引力(重力)皆為保守力

$$\Rightarrow \quad F_G = \frac{-dU}{dr}, \quad U_G = -\int \overrightarrow{F_G} \cdot d\vec{r}$$

重力 $F_G = \frac{Gm_1m_2}{r^2}, \quad U_G = -\int \overrightarrow{F_G} \cdot d\vec{r} = -\frac{Gm_1m_2}{r}$ [引力位能為負]

電力 $F_{客} = \frac{kQ_{主}q_{客}}{r^2}, \quad U_{客} = -\int \vec{F}_{客} \cdot d\vec{r} = \frac{kQ_{主}q_{客}}{r}$ [斥力位能為正]

球狀等位面 (*Equipotential Surface*) →球面上任一點電位皆相等



$$V_1 < V_2 < V_3$$

$$Q_{主} \rightarrow \vec{E}_{主} = \frac{kQ_{主}}{r^2} \rightarrow V_{主} = \frac{kQ_{主}}{r}$$

$$q_{客} \rightarrow \vec{F}_{客} = q_{客}\vec{E}_{主} = \frac{kQ_{主}}{r^2}$$

$$\rightarrow U_{客} = q_{客}V_{主} = \frac{kQ_{主}}{r}q_{客}$$

Perpendicular

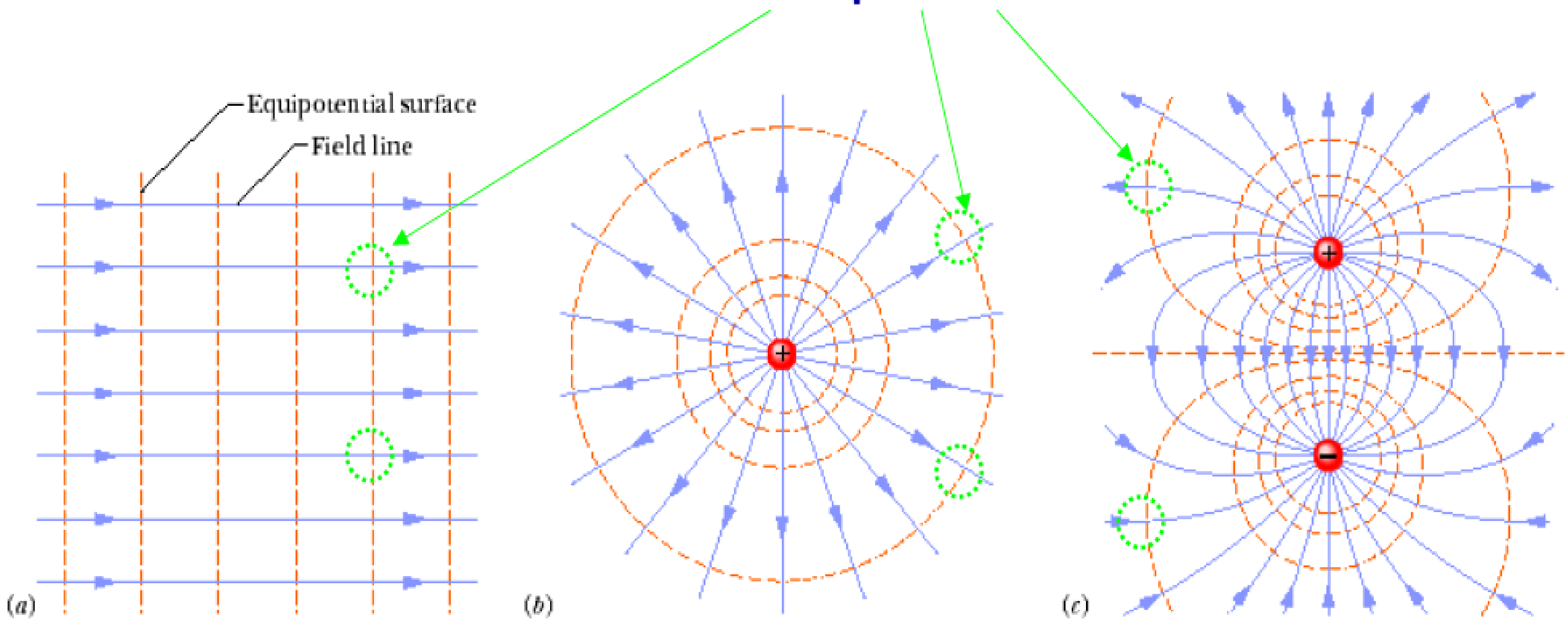


Fig. 24-3 Electric field lines (purple) and cross sections of equipotential surfaces (gold) for (*a*) a uniform electric field, (*b*) the field due to a point charge, and (*c*) the field due to an electric dipole.

6. 電偶(Electric Dipole)的中心軸上任一點電場與電位

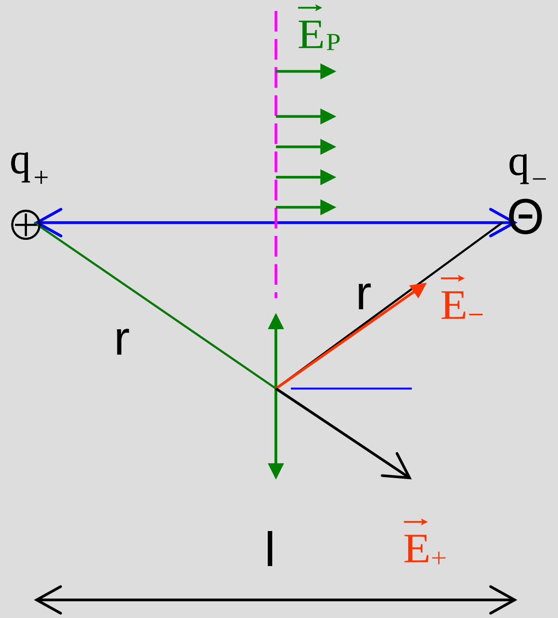
電偶→同電量之正、負電荷隔著一小段有效距離

$$|q_+| = |q_-|$$

中心軸上P點的電場大小

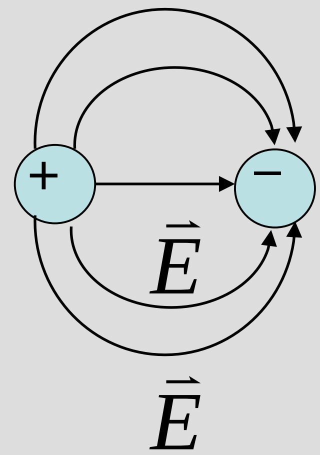
$$\vec{E}_P = \vec{E}_+ + \vec{E}_- \quad \left[|\vec{E}_+| = |\vec{E}_-| = \frac{kq+}{r^2} \right]$$

$\vec{E}_{+y}$ and $\vec{E}_{-y}$ 方向相反抵消



$$\Rightarrow \vec{E}_P = \vec{E}_{+X} + \vec{E}_{-X} = E_+ \cos\theta + E_- \cos\theta = 2\vec{E}_+ \cos\theta$$

$$= 2\frac{kq_+}{r^2} \times \frac{l}{2r} = \frac{kq_+}{r^3} l$$



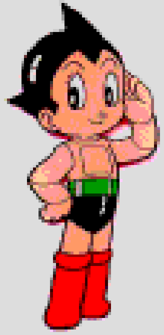
$$V_p = V_+ + V_- = \frac{kq_+}{r} + \frac{kq_-}{r} = 0$$

$q_+ = -q_-$ $\Rightarrow$ 中心軸為電位為零(等位面)

例題10. 某點電荷 $q_1 = 4 \times 10^{-8} C$ 在距離2cm處之電場及電位?

若有另一點電荷 $q_2 = -4 \times 10^{-10} C$ 在離 q_1 4cm處所受的庫倫力及電位能?

請回家練習!



例題11.電偶電量 $8\times10^{-9}C$ 長18 cm，求在中心軸上離中心12 cm 處

(1) 此點之電場及電位

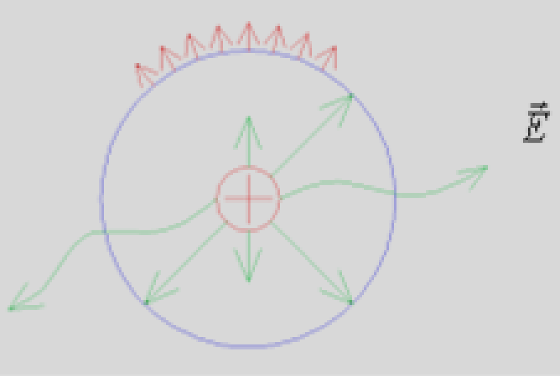
(2)若 $q_{客}=6.4\times10^{-11}C$ 求所受的電力及電位能

請回家練習！

7. 電場高斯定律 (Gauss Law)

封閉空間內含有電量Q，所散發出電場之電力線穿透整個封閉表面之總電力線數(電通量 Φ_E) 為 $4\pi k Q$

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = 4\pi k \cdot Q = \frac{Q}{\varepsilon_0}$$



電通量(*Electric Flux*) = 垂直通過截面之有效電力線數

$d\vec{\Phi}_E = \vec{E} \cdot d\vec{A} =$ 有效垂直截面之電力線數

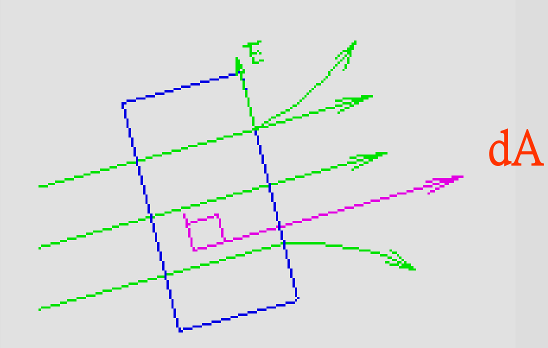
電通量

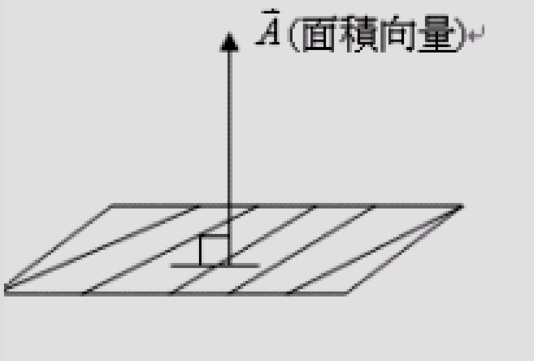
$$\vec{E}_\perp = E \cdot \cos\theta$$

$$d\Phi = \vec{E} \cdot d\vec{A}$$

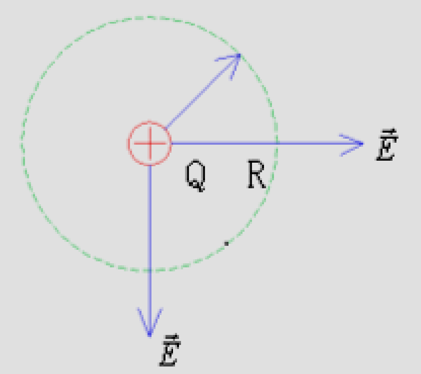
$$= \vec{E}_\perp \cdot d\vec{A}$$

$$= \left|\vec{E}_\perp\right| \cos\theta \left|d\vec{A}\right|$$





整個封閉表面之總電通量 $\Phi = \oiint d\Phi = \oiint E \cdot d\vec{A}$



找到封閉球面，半徑為R球面上電場 $\vec{E}=\frac{kQ}{R^2}$ 通過封閉球面上的總電通量。

$$\Phi = \oiint d\Phi = \oiint \vec{E}\cdot d\vec{A}$$

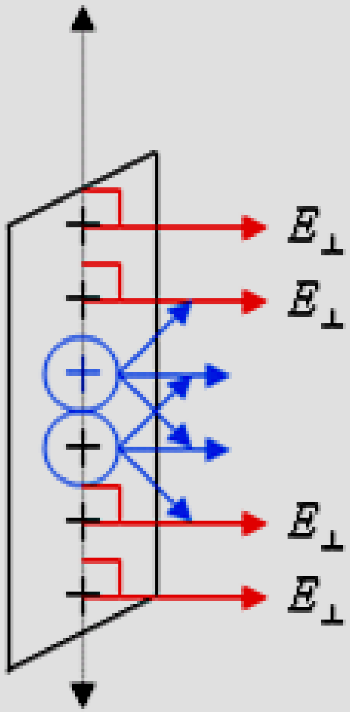
$\vec{\mathbf{E}}$和$\mathbf{d}\vec{\mathbf{A}}$是平行的 $\theta = 0^{\circ} \Rightarrow \cos\theta = 1$

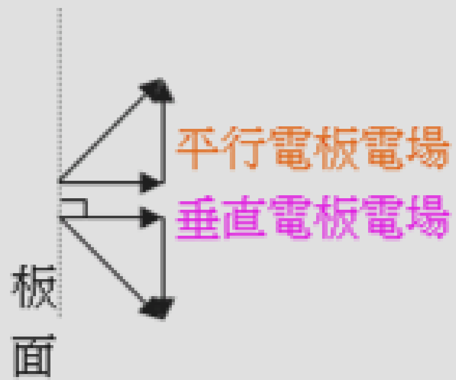
$$\Phi_E = \oiint \frac{kQ}{R^2} d\vec{A} = \frac{kQ}{R^2}\oiint d\vec{A} = \frac{kQ}{R^2}\times 4\pi R^2 = 4\pi kQ$$

8. 均勻帶電之電板電場

→ 均勻帶電 → $\sigma = \dfrac{Q}{A} = \dfrac{dQ}{dA} = Constant$

無限長電板之電場特性→ $\dfrac{\text{上}}{\text{下}}$ 鄰電荷平行電之電場相互抵消，只剩下垂直電板之電場



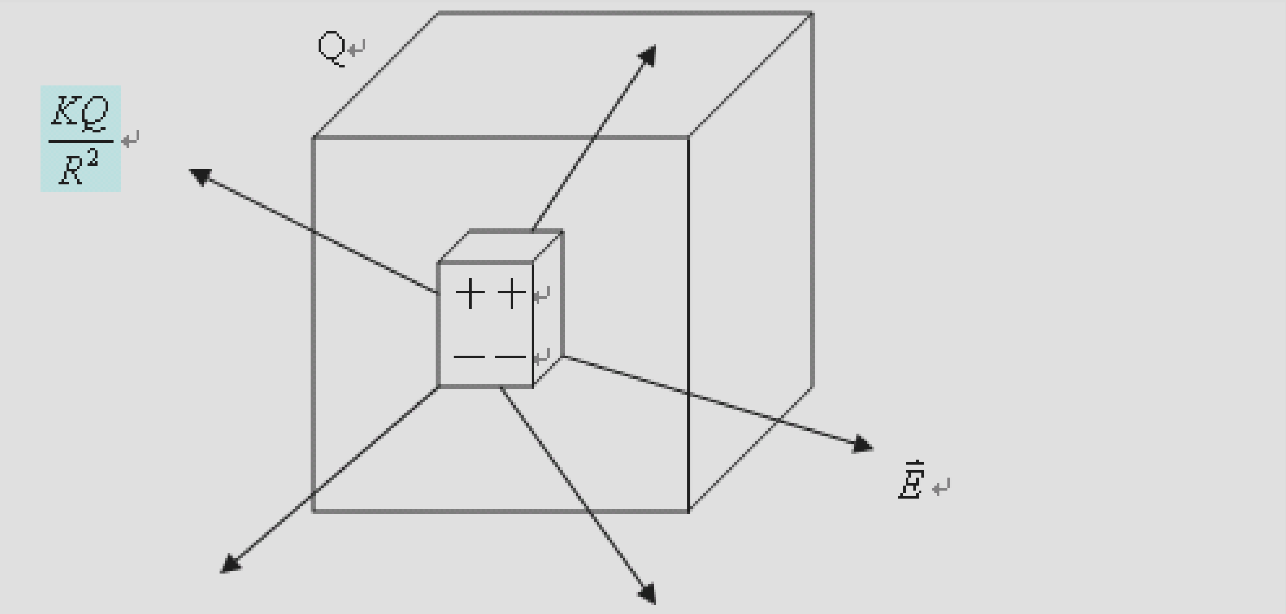


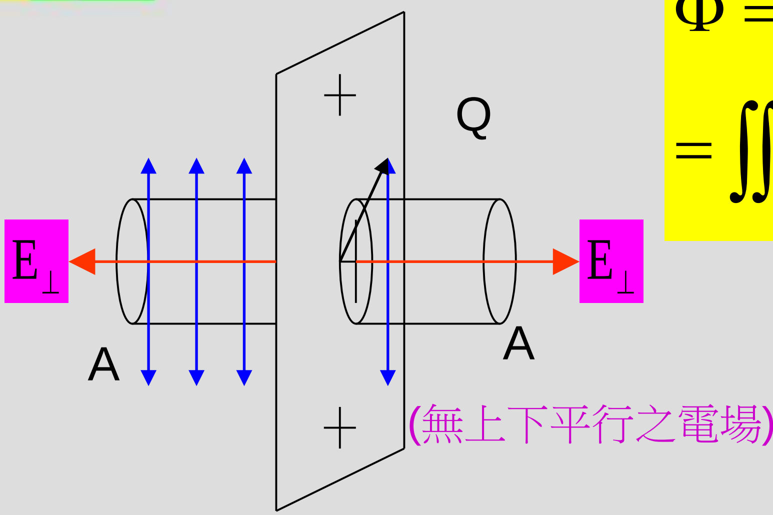
因此電板之有效電場→垂直電板之電場($E_{\perp}$)

找一對稱圓柱密閉空間→利用電場之高斯定律

※ 高斯定律可適用於求任何對稱形狀之帶電物體之電場

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = 4\pi k \cdot Q = \frac{Q}{\varepsilon_0}$$





$$\Phi = \oiint d\Phi = \oiint \vec{E}\cdot d\vec{A}$$

$$= \iint_{左面} \vec{E}\cdot d\vec{A} + \iint_{右面} \vec{E}\cdot d\vec{A} + \iint_{柱面} \vec{E}\cdot d\vec{A}$$

$$= E_{\perp}\cdot A + E_{\perp}\cdot A$$

$$= 2E_{\perp}\cdot A = 4\pi k\cdot Q$$

$$\frac{Q}{A} = \sigma = Cons\,tan\,t$$

$$= \text{均匀}$$

$$※\ E_{\perp} = 2\pi k\cdot\frac{Q}{A}$$

$$= 2\pi k\cdot\sigma = \text{常數} = Cons\,tan\,t$$

（σ ：表面電荷密度

為何無限長帶電板電場和距離無關

→ P點離電板近，距離小，各電荷貢獻電場強度強

上/下 遠方的電荷貢獻電場幾乎平行電板電場已互相抵消，無效應

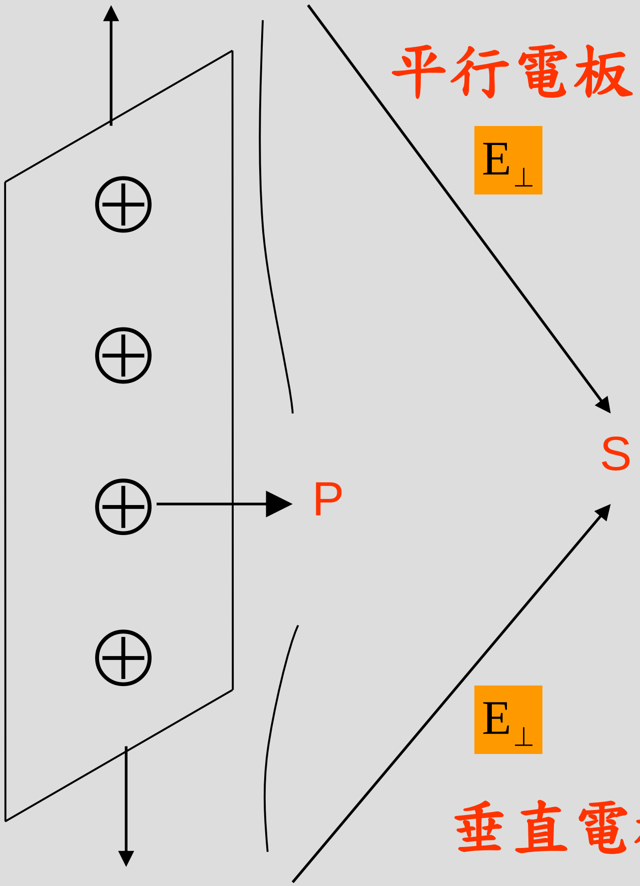
距離近 { 電場強度強 / 有效電荷少 }

→ S點離電板遠，距離大，各電荷貢獻電場強度弱

上/下 遠方的電荷貢獻電場仍含有垂直電板之有效電場未能相互抵消，有效應

距離遠 { 電場強度弱 / 有效電荷多 }

平行電板電場，以抵消



$$E_{\perp} = 2\pi k \cdot \sigma$$

垂直電板之電場

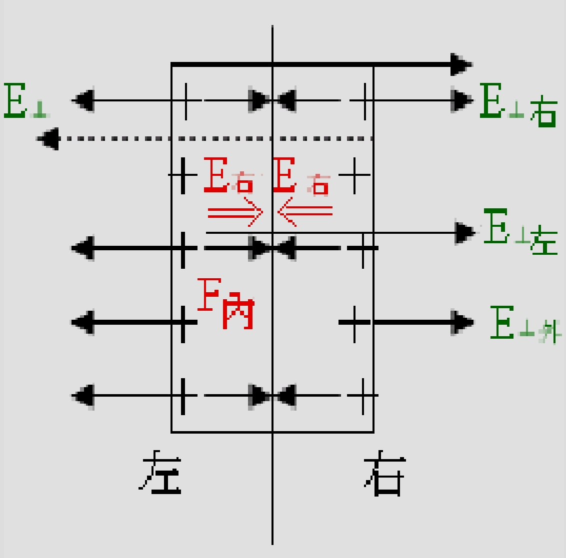
$$\vec{E}_P = \sum_i \frac{kq_i}{r^2} \qquad i < j \quad near$$

$$\vec{E}_S = \sum_j \frac{kq_j}{r_j^{\,2}} \qquad r_i < r_j \quad far$$

$$\Rightarrow \quad \vec{E}_P = \vec{E}_S$$

9. 均勻帶電導體之內外部電場

⇒ 導體帶電 ⇒ 電荷皆分佈在表面上
以金屬板塊為例 → 電荷均勻分佈在兩側



以近似均勻帶電無限電板之電場($E_{\perp} = 2\pi k\sigma$)

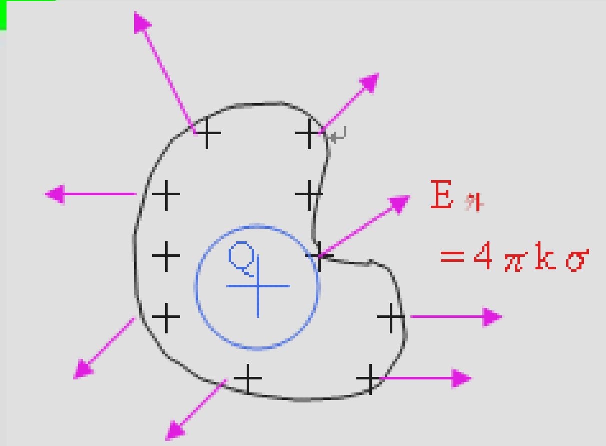
內部→ $E_{內} = E_{左} + E_{右}$

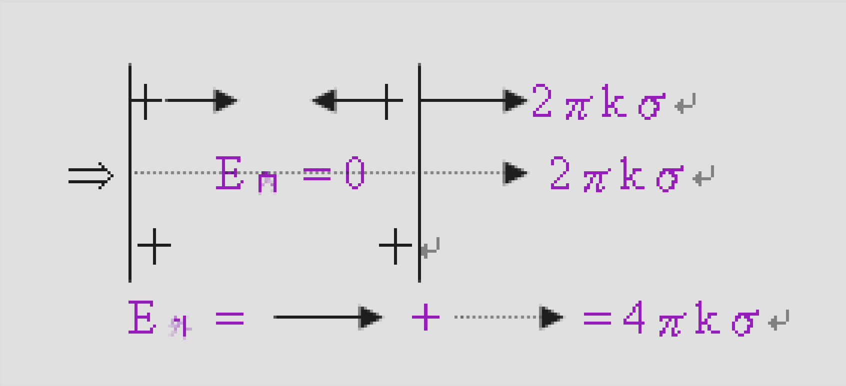
$= 0$ （電場方向相反）

外部→ $E_{外} = E_{\perp(左)} + E_{\perp(右)}$

$= 4\pi k\sigma$（電場方向相同）

⇒ 延伸至任何形狀帶電的導體之內外部電場





導體內部電場 $E_{內} = 0$

【可用高斯定律說明→因電荷分佈在表面，內部無電荷】

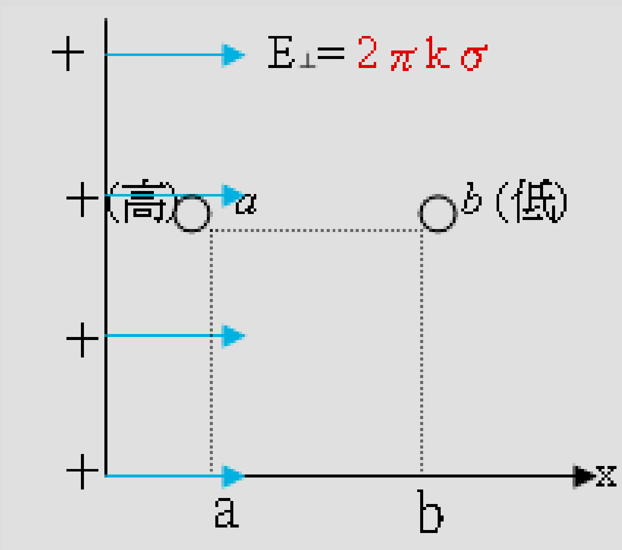
$$\Rightarrow \Phi = \oiint \overrightarrow{E_{內}} \cdot d\overrightarrow{A} = 4\pi k Q_{內}, \quad Q_{內} = 0 \rightarrow \overrightarrow{E_{內}} = 0$$

導體外部電場 $E_{外} = 4\pi k\sigma$ （$E_{外}$是垂直於表面）

10. 均勻電場之電位及電位差

$\Rightarrow$ 均勻電場 $E = Constant$

以均勻帶電無限長電板之電場 $E = 2\pi k\sigma = Constant$



（庫侖定律）

↑

點電荷

$$F_{客} = \frac{kq_{主}q_{客}}{r^2}$$

$$E_{主} = \frac{F_{客}}{q_{客}}$$

$$= \frac{kQ_{主}}{r^2}$$

$$U_e = -\int \vec{F}_e \cdot d\vec{r}$$

↑

$$q_{主} \rightarrow E_{主} = \frac{kq}{r^2} \rightarrow V_{主} = \frac{kQ_{主}}{r}$$

$$U_{客} = -\int \vec{F}_{客} \cdot d\vec{r} = \frac{kQ_{主}q_{客}}{r}$$

$$V_{主} = \frac{U_{客}}{q_{客}} = \frac{kQ_{主}}{r}$$

$$-\int \frac{\vec{F}_{客}}{q_{客}} \cdot d\vec{r} = -\int \vec{E}_{主} \cdot d\vec{r}$$

$$V_{主} = -\int E_{主} \cdot dr$$

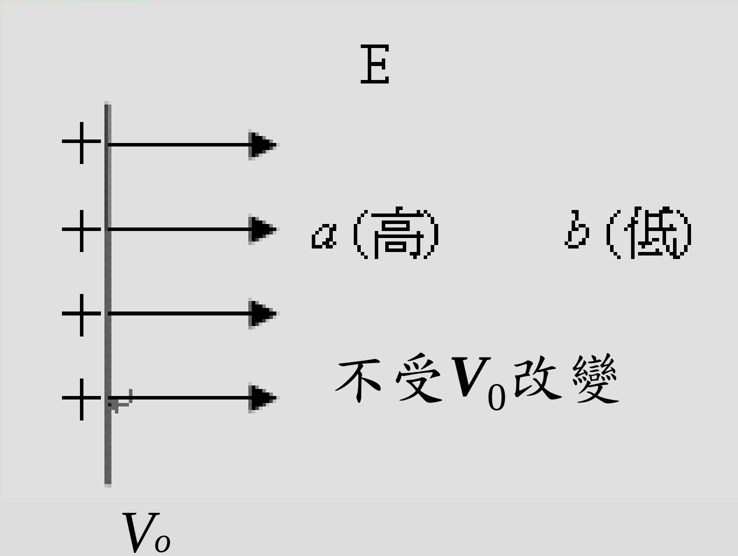
$$V(x) = -\int E \cdot dx$$

$$V(a) - V(0) = -\int_0^a E \cdot dx$$

$$V(0 \to a) = V_a - V_0 = V(a) - V(0)$$

$$= -E \cdot \int_0^a dx = -E \cdot x \Big|_0^a = -E(a-0) = -Ea < 0$$

$$\Rightarrow \quad V_a < V_0$$



$$
\begin{cases}
V_a - V_0 = -E \cdot (a - 0) \\
V_a = V_0 - E \times a \\
V_b = V_0 - E \times b
\end{cases}
$$

$V_a > V_b$ $\Rightarrow$ a 離正電荷近 $\rightarrow$ 電位高，反之

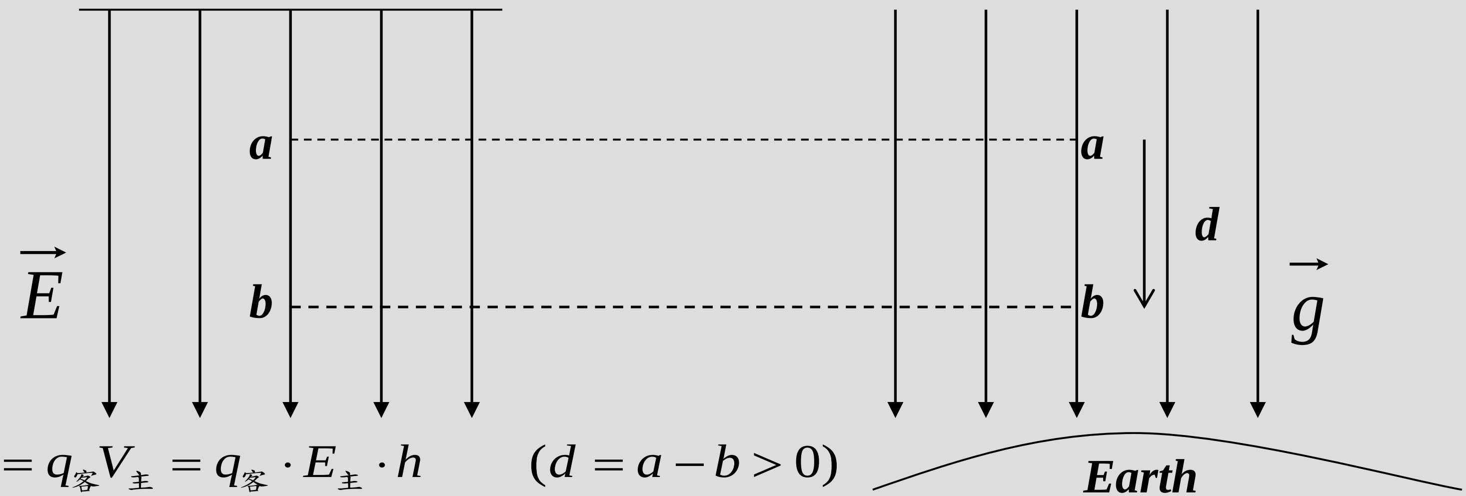
$(a < b)$ b 離正電荷遠 $\rightarrow$ 電位低

$\Rightarrow$ b 離負電荷近 $\rightarrow$ 電位低，反之

a 離負電荷遠 $\rightarrow$ 電位高

$$\vec{F}_{客} = q_{客}\vec{E}_{主} \qquad\qquad \vec{F}_{客} = m_{客}\vec{g}_{主}$$

電場　　　　　　　　重力場



$$U = q_{客}V_{主} = q_{客}\cdot E_{主}\cdot h \qquad (d = a - b > 0)$$

$$U_{a\to b} = U_b - U_a = 低 - 高$$
$$= q\cdot E\cdot (b-a)$$
$$= q\cdot E\cdot (-d) < 0$$

$$U = m_{客}\cdot g_{主}\cdot h$$

$$U_{a\to b} = U_b - U_a = 低 - 高$$
$$= m_{客}\cdot g_{主}\cdot (b-a) \qquad \textbf{(d：間距)}$$
$$= m_{客}\cdot g_{主}\cdot (-d) < 0$$

★ 正電荷 → 傾向由 高電位 流至 低電位

高電位能 流至 低電位能

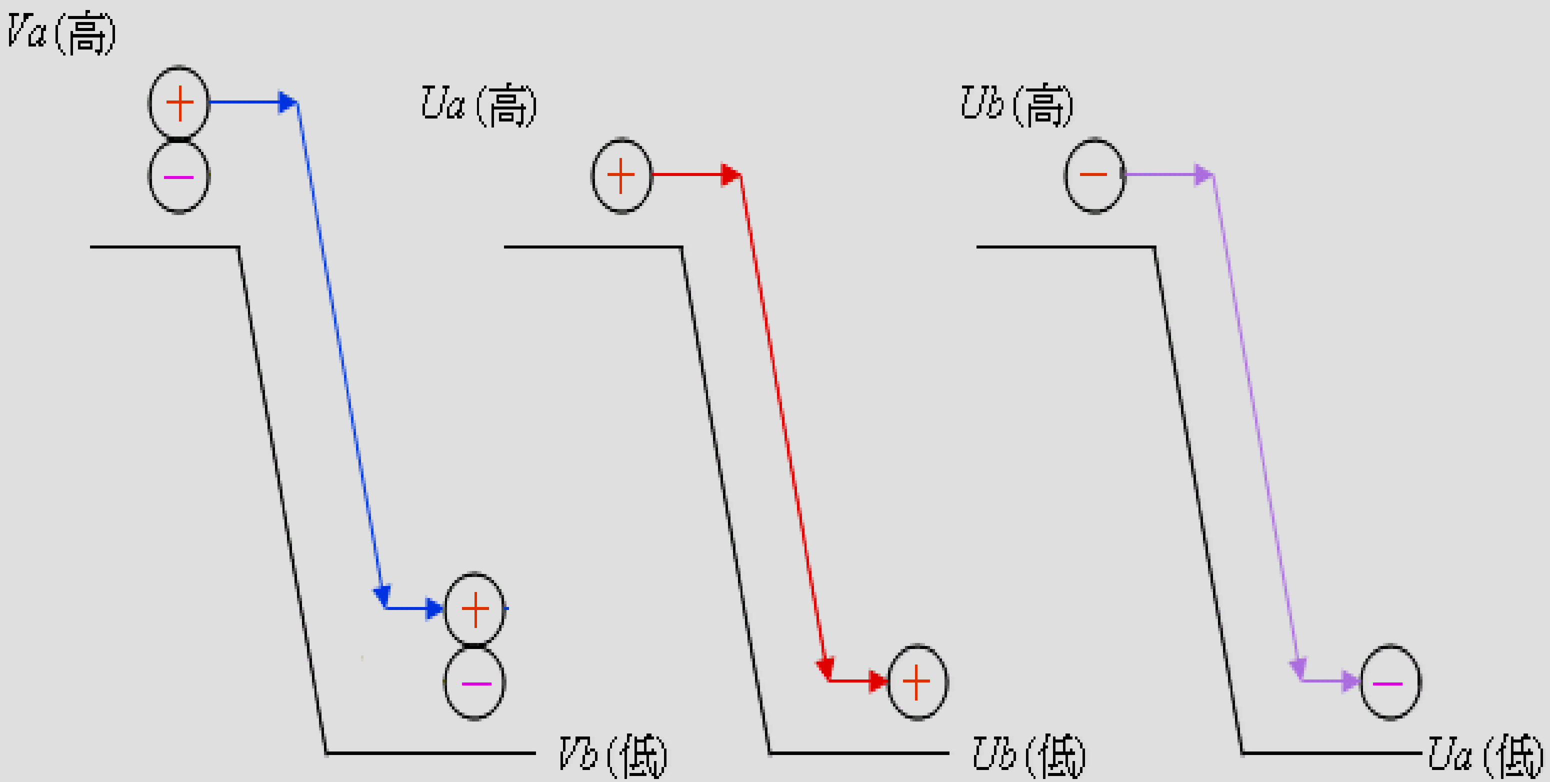
負電荷 → 傾向由 低電位 流至 高電位

高電位能 流至 低電位能

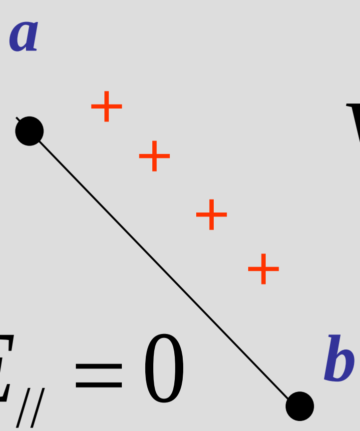
$$E = U + K$$

(位能減少→動能增加，正功)

(位能增加→動能減少，負功)


Va (高)
Ua (高)
Ub (高)
Vb (低)
Ub (低)
Ua (低)

為何導體表面是等位面？



$$\Rightarrow$$ 利用**電位差**和**電場垂直導體表面**

$$V_b - V_a = E \cdot d = E_{//} \cdot d = 0$$

($E_{//}$ = 0 平行表面電場)

$$E_{//} = 0$$

$$V_a = V_b$$

（表面任何兩點電位相等，所以導體表面為相等電位）

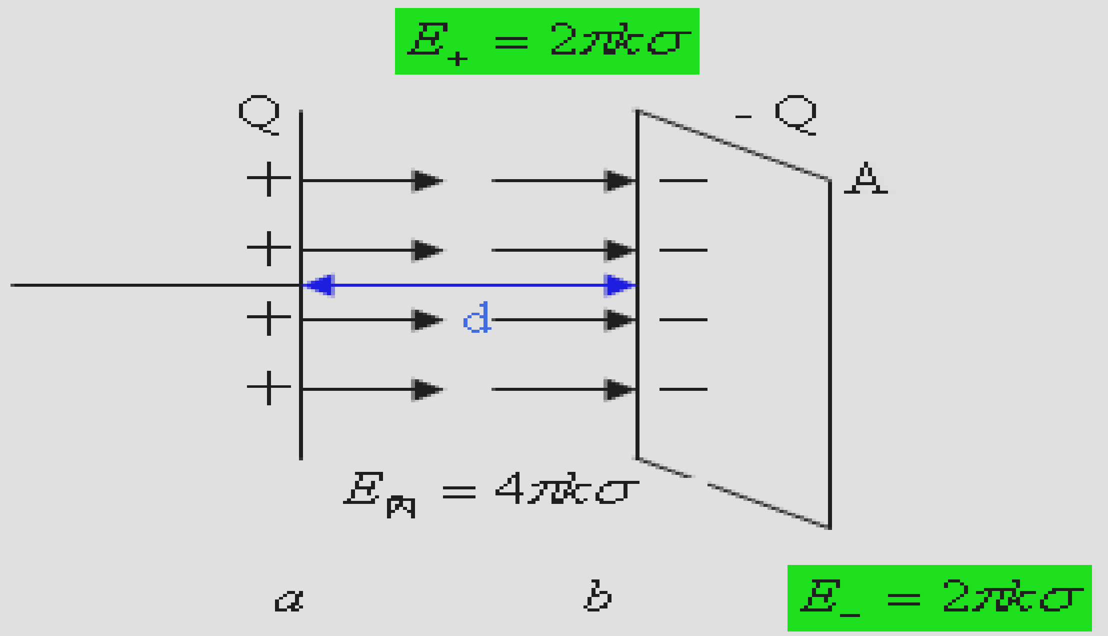
例12. 一無限長均勻帶電板，一正電荷 $q=3\times10^{-6}C$ 距離電板 30cm處受到電力5N，求電板之電場和表面電荷密度？此正電荷移至50cm處，電位差？電位能的變化量？

請回家練習！

11.電容 (Capacitor)

⇒ 儲存電荷(Q)的元件

⇒ 利用均勻帶電無限長電板之電場 ($E = 2\pi k \cdot \sigma$)



電容內部電場 $E_{內} = E_{+} + E_{-} = 4\pi k\sigma = 4\pi k\frac{Q}{A}$ （$E_{+} = E_{-} = 2\pi k\sigma$）

電容內部電位 $\Delta V = V_a - V_b = E_{內}\cdot d = 4\pi k\sigma\cdot d$

儲存電荷Q多$\rightarrow$ $E_{內} = 4\pi k\frac{Q}{A}$ **大**

$V = E_{內}\cdot d = 4\pi k\frac{Q}{A}\cdot d$ **高** （**電位差越大**）

$Q \propto E_{內} \propto V \rightarrow$ $\mathbf{Q = CV}$

電容的定義：$C = \frac{Q}{V} = \frac{Q}{4\pi k \cdot \frac{Q}{A} \cdot d} = \frac{1}{4\pi k} \cdot \frac{A}{d}$

$$= \varepsilon_o \cdot \frac{A}{d}$$

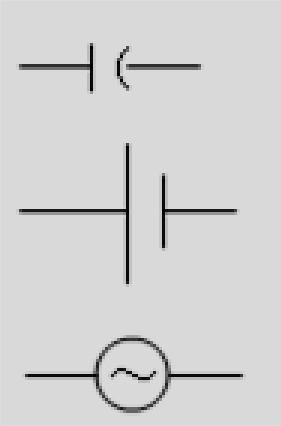
$\left\{\begin{array}{l}\text{和電板面積(A)成正比} \\ \text{和電板距離(d)成反比}\end{array}\right\}$

$\varepsilon_o = \frac{1}{4\pi k}$ **= 8.85 × 10^{-12} C^2/N-m^2**

允許率　　　　真空

【Permittivity of free space】

電容電荷之獲得，須從外界充電

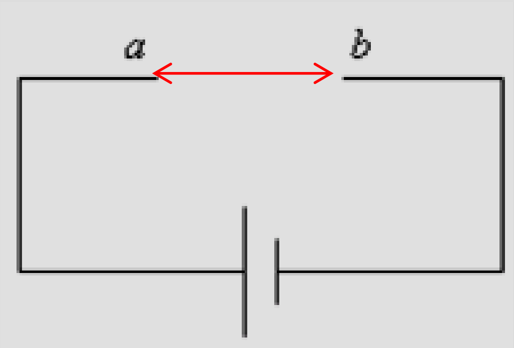


電容在電路上的圖示

直流電池在電路上的圖示

交流電源在電路上的圖示

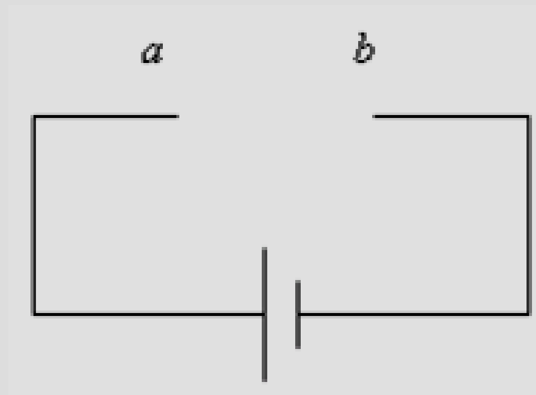
短路 (Short-Circuit) →



$V_{ab} = 0$

$I_{ab} \neq 0$

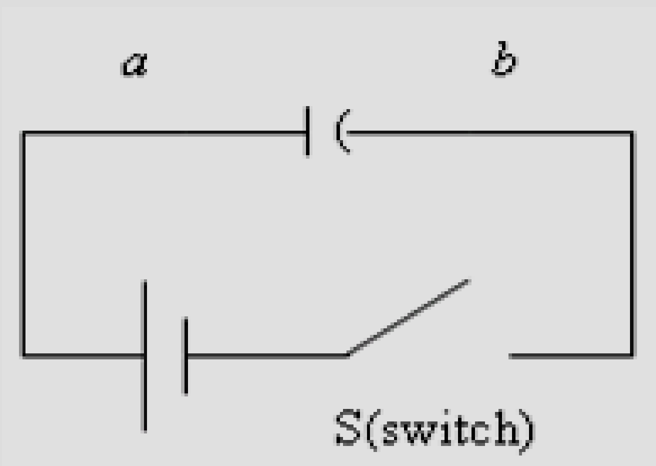
開路 (Open-Circuit) →



$I_{ab} = 0$

$V_{ab} \neq 0$

電容充電前 ($t = 0_-$)



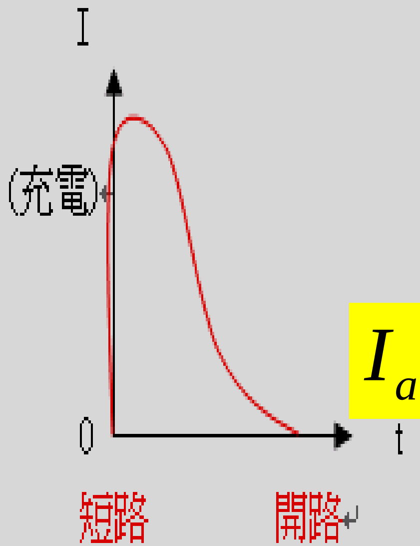
$Q = 0$

$E = 4\pi k\sigma = 4\pi k \cdot \frac{Q}{A} = 0$

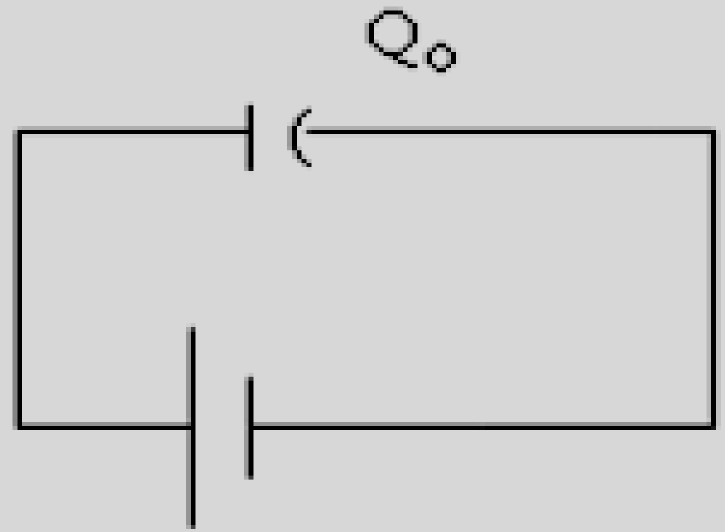
$V = E \cdot d = 0$ （充電瞬間為短路）

$U(t=0) = QV = 0$

電容充電完後 ($t = \infty$)



$$I_{ab} = 0$$



$$Q = Q_O$$

$$E = 4\pi k \cdot \sigma$$

$$V = E \cdot d = E_O \cdot d = V_O$$

$$U(t = \infty) = Q_O V_O$$

充電後電容視為開路 $I_{ab} = 0$

電路之電位能，須經充電時間的累積

其電位能應以**平均值觀點**來定之

$$\overline{U}_e = \frac{U(t=0)+U(t=\infty)}{2} = \frac{1}{2}Q_oV_o$$

$$= \frac{1}{2}C_oV_o^{\ 2} \qquad (Q_o = C_oV_o)$$

$$= \frac{1}{2}\frac{Q_o^{\ 2}}{C_o} \qquad (V_o = \frac{Q_o}{C_o})$$

☆☆ 能量密度【$\mu_e = \frac{\overline{U}_e}{V}$ = 電位能/電容體積】

$$\mu_e = \frac{U_e}{V} = \frac{\frac{1}{2}Q_oV_o}{A \cdot d}$$

$$= \frac{\frac{1}{2}C_oV_o^{\;2}}{A \cdot d} = \frac{\frac{1}{2}(\frac{\varepsilon_o A}{d})(E_o \cdot d)^2}{A \cdot d} = \frac{1}{2}\varepsilon_o E_o^{\;2}$$

$\Longrightarrow$ { 能量密度與$E_o^{\;2}$成正比
電場能量密度 }

Advanced Technology & Integrated Systems Laboratory

電容系統之物理量

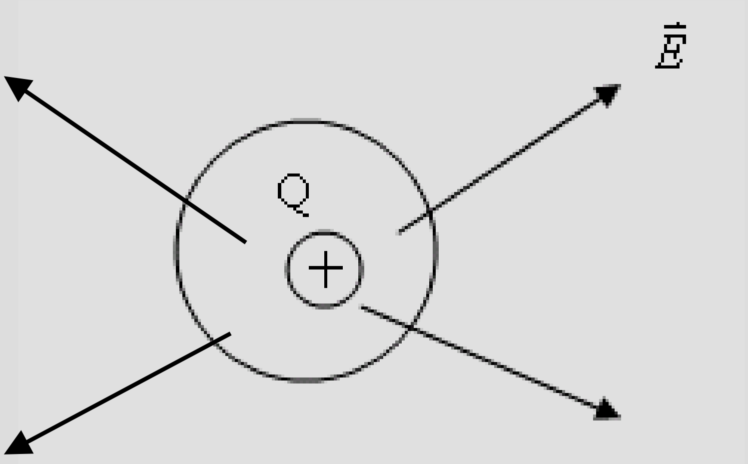
$$(Q_o \quad E_o \quad V_o \quad U_e \quad \mu_e \quad C_o)$$

$$\left(Q_o \quad 4\pi k\frac{Q_o}{A},\ E_o \cdot d,\ \frac{1}{2}Q_oV_o,\ \frac{1}{2}\varepsilon_o E_o^{\ 2},\ \frac{\varepsilon_o A}{d} \right)$$

$$\varepsilon_o = \frac{1}{4\pi k}$$

$$\Phi = \oiint \vec{E} \cdot d\vec{A} = 4\pi k \cdot Q = \frac{Q}{\varepsilon_o}$$

ε_o 電場電力線穿透真空係數(允許)



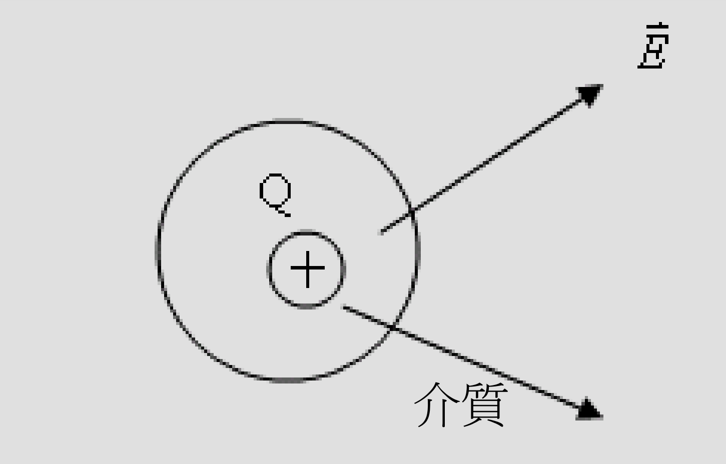
係數大，穿透率小

係數小，穿透率大

$$\Phi = \oiint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon} = \frac{Q}{\kappa \varepsilon_0}$$

（κ：介電常數 ＞ 1 ）

$\varepsilon \rightarrow$ 電場電力線穿透介質的係數



電磁波之電場能量密度和磁場能量密度是相等而可互相轉換

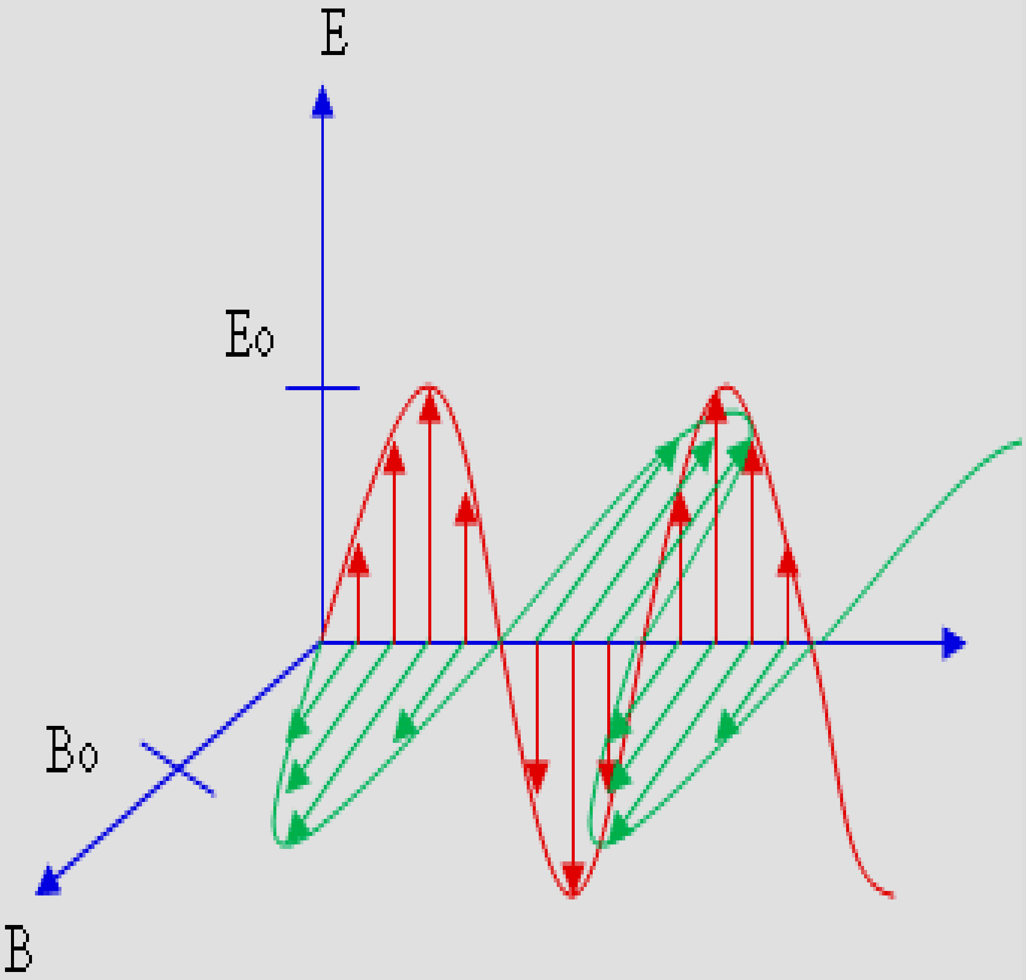
$$\mu_E = \mu_B$$

$$\frac{1}{2}\varepsilon_o E_o^{\ 2} = \frac{1}{2}\frac{1}{\mu_o}B_o^{\ 2} \Rightarrow B_o^{\ 2}\cdot\frac{1}{\mu_o\varepsilon_o} = E_o^{\ 2}$$

【 $\frac{1}{\mu_o\varepsilon_o} = C^2$ **(光速 C = 3 × 10⁸ m/s)**】

$$B_o^{\ 2}\cdot C^2 = E_o^{\ 2}$$

$$\Rightarrow \quad B_o \cdot C = E_o$$



$$E(t) = E_0 \cdot \sin(\omega t)$$

$$B(t) = B_0 \cdot \sin(\omega t)$$

$\vec{S}$：電磁波進行方向

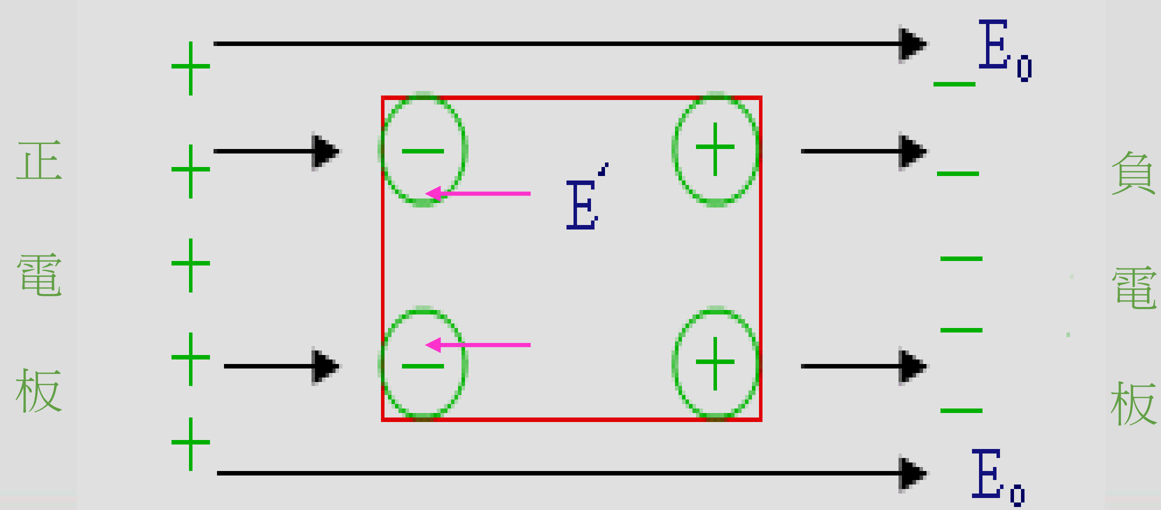
$$\vec{E} \perp \vec{B}$$

$$\vec{S} = \vec{E} \times \vec{B}$$

12. 電容放大器

⇒ 插入介電質於電容中，電容值增加 κ 倍【κ：介電常數 > 1】
(Dielectrics) (Dielectric constant)

介電質：置入電場中，部份表面可被極化產生極化電荷之絕緣體

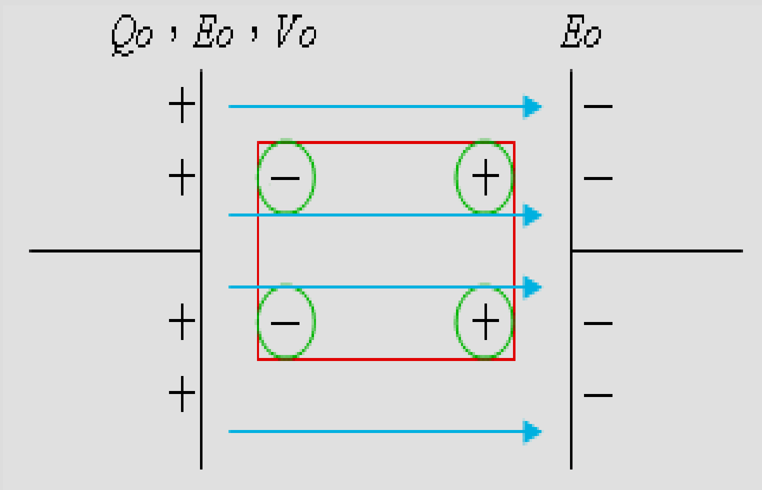


未插入介電質，電容系統之物理量

$(Q_o，E_o，V_o，U_{eo}，\mu_{E.O}，C_o)$

插入介電質 $(Q, E, V, U_e，\mu_E，C)$

- 獨立電容



$\vec{E} = \vec{E}_o + \vec{E}_{極化}$ 小

$V = E \cdot d$ 小

$C = \frac{Q}{V}(小)$ 大

電容電板之場在介電質表面極化出少許部份的極化電荷，

此極化電荷在介電質內部形成極化電場

$$\vec{E}_{極化} \quad (\vec{E}_{極化} = -(1-\frac{1}{\kappa})E_o)$$

和原來電場E_o方向相反，因此減弱了電容之總電場E

$$\vec{E} = \overrightarrow{E_o} + \vec{E}_{極化} = \overrightarrow{E_o} - (1-\frac{1}{\kappa})\overrightarrow{E_o} = \frac{1}{\kappa}\overrightarrow{E_o}$$

$$V = E \cdot d = \frac{1}{\kappa}E_o \cdot d = \frac{1}{\kappa}V_o$$

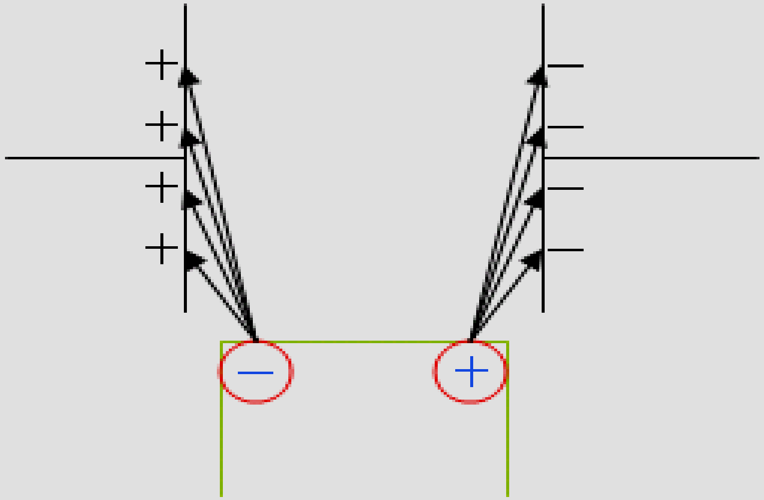
$$C = \frac{Q}{V} = \frac{Q = Q_0}{(\text{電荷無處可逃})} \frac{Q_0}{\frac{1}{\kappa}V_0} = \kappa\frac{Q_0}{V_0} = \kappa C_0$$

【κ> 1 電容值放大為κ倍】

$$U_E = \frac{1}{2}QV = \frac{1}{2}Q_o\frac{V_o}{\kappa} = \frac{U_{e.o}}{\kappa}$$

$$\mu_E = \frac{U_E}{V(\text{電容體積不變})} = \frac{\frac{1}{\kappa}U_{eo}}{V_o} = \frac{1}{\kappa}\mu_{E.o}$$

電位能減少⇨ 部份電位能加速至介電質進入的動能增加

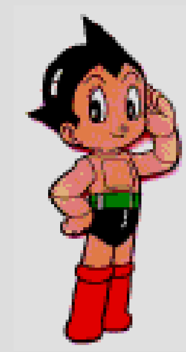


加速吸入

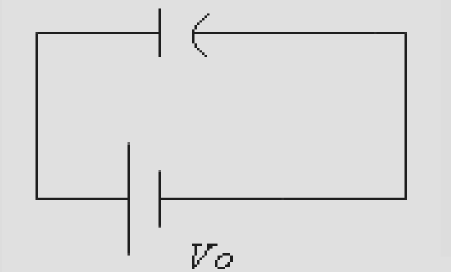
(介電質進入吸力增大，速度增快→動能增加)

例13. 一獨立電容被充至10V，其電板截面積為200 cm^3, 電板距離為10 cm，電容值?離電板3 cm之電位差?所儲存電荷?若插入介電質，其極化電場為原來電場2/9倍，求(Q、E、V、U_E、μ_E、C)為原來各幾倍?κ=?

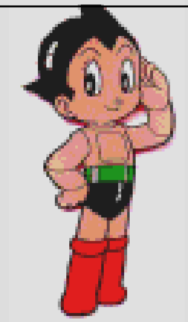
請回家練習!



例14. 電容接上一固定電池，插入介電質($\kappa = 3$)，求(Q, E, V, U_e, μ_E, C)為原來的幾倍？

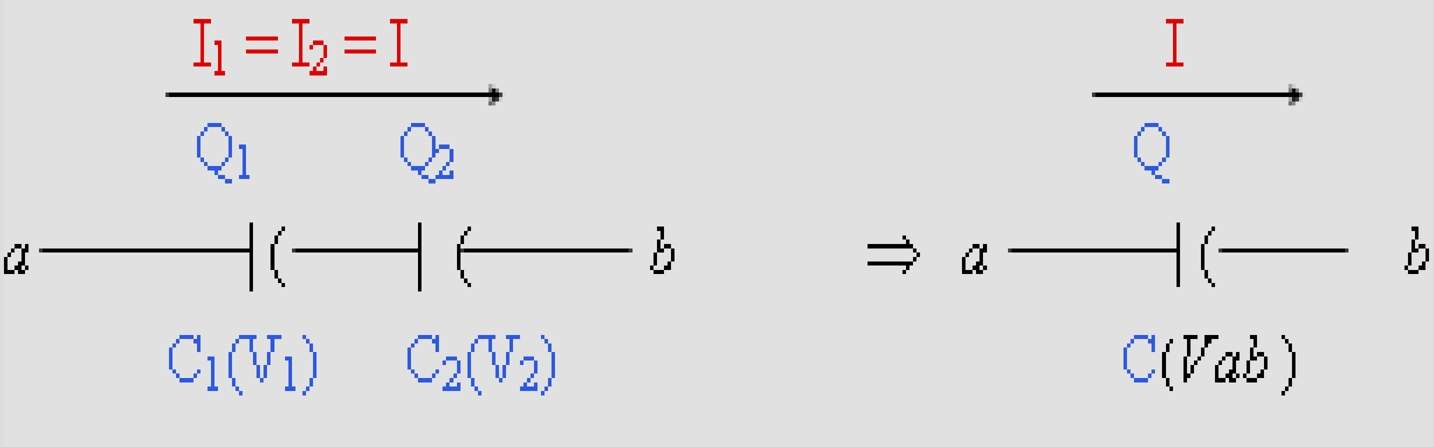


請回家練習!



13. 電容之串聯與並聯

串聯(SERIES CONNECTION)



串聯→流經兩元件之電流相等（ $I_1 = I_2 = I$ ）
電壓(位)為兩元件電壓之總和

$$V_{ab} = V_1 + V_2$$

電流 $I = \dfrac{\Delta Q}{\Delta t}$ (單位時間流過電量)　$I \propto Q$

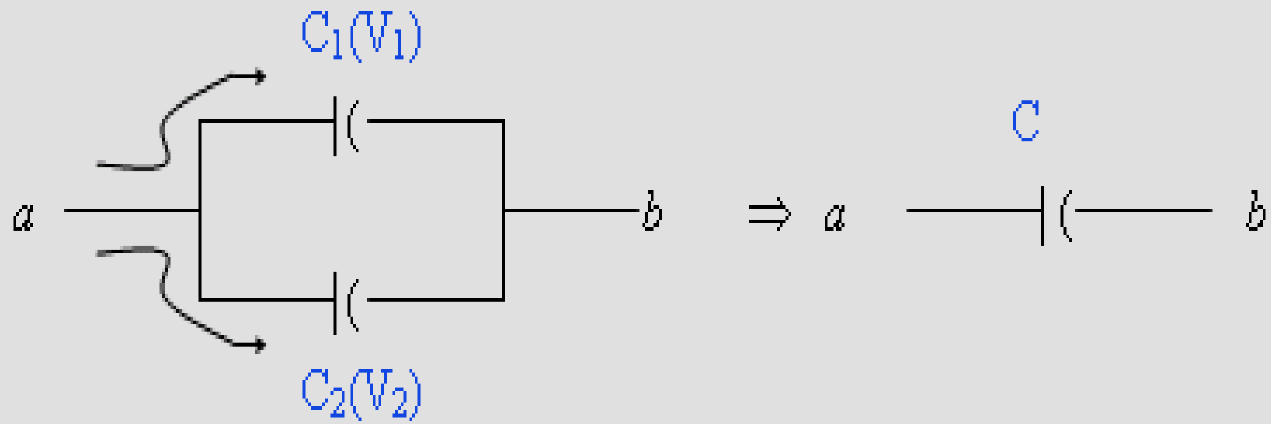
流經電荷皆相等 $\Rightarrow$ $Q_1 = Q_2 = Q$

利用 $Q = CV \Rightarrow V = \dfrac{Q}{C}$ 代入

$$V_{ab} = \frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

→ 並聯 (PARALLEL CONNECTION) 平行



並聯→跨在各個元件之電壓相等($V_1 = V_2 = V_{ab}$)

電流為兩元件之電流總和 ($I \propto Q$)

$$Q = Q_1 + Q_2 = C_1V_1 + C_2V_2 = CV_{ab}$$

$$= C_1V_{ab} + C_2V_{ab} = CV_{ab}$$

$$= (C_1 + C_2)V_{ab}$$

$$C_1 + C_2 = C$$