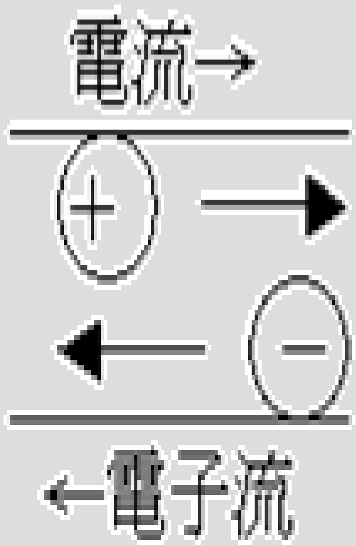
Chapter 8. 電流 Electric Current

1. 電流 (Electric Current)

→ 電流是指正電荷移動

電子流是指負電子移動



一般而言，導線中並無正電荷傳導移動

只有電子在移動傳導

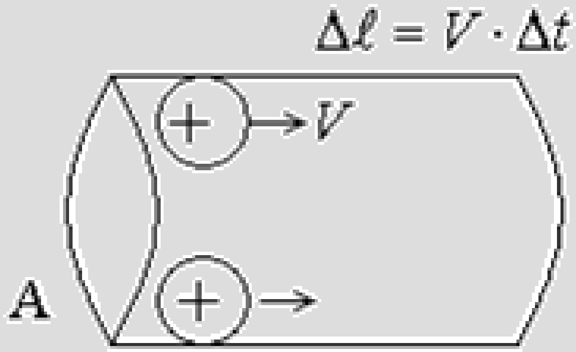
電流→電子流之反向

電流之巨觀定義 $I = \dfrac{\Delta Q}{\Delta t}$ (單位時間內流過截面之電量)

電流是純量 【C/s = Amper (安培)】

電流的微觀定義

$$I=\frac{\Delta Q}{\Delta t}=\frac{(\text{流過電荷數})\cdot q}{\Delta t}=n\cdot A\cdot V\cdot q$$



A→導線截面積
V→電荷的移動速度
N→導線中電荷濃度
(單位體積的電荷數，# /m^3)

$$\text{流過電荷數}=n\cdot\Delta V$$
$$=n\cdot A\cdot\Delta\ell$$
$$=n\cdot A\cdot V\cdot\Delta t$$

n : 不同金屬 (Cu、Al、Ag….) → 查表

A :導線管徑面積

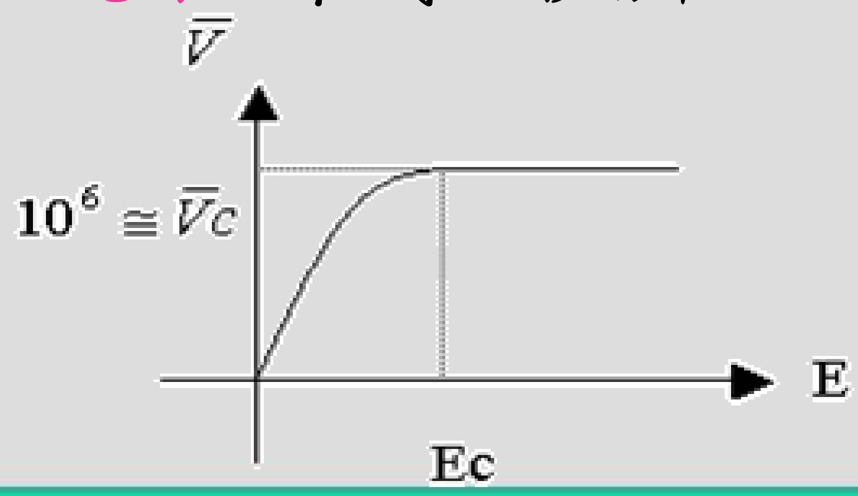
$$V = V_{rms} = \sqrt{\frac{3k_B T}{m}}$$

V :電子平均移動速度 → 溫度 (電場效應弱)

$$= 1.16 \times 10^5 \, m/s$$

↘

電場 $V = \mu E$ (μ = 移動率，Mobility)



2. 歐姆定律之微觀及巨觀公式

→ 微觀公式 $J = \sigma E$

巨觀公式 $I = \dfrac{V}{R}$

☆ 微觀公式之推導

$$J\,(\text{電流表面密度}) = \frac{I}{A}$$

$$\Rightarrow J = \frac{I}{A} = \frac{n \cdot A \cdot V \cdot q}{A} = n \cdot V \cdot q$$【電場效應$E \xrightarrow{\text{主宰}}$電流】

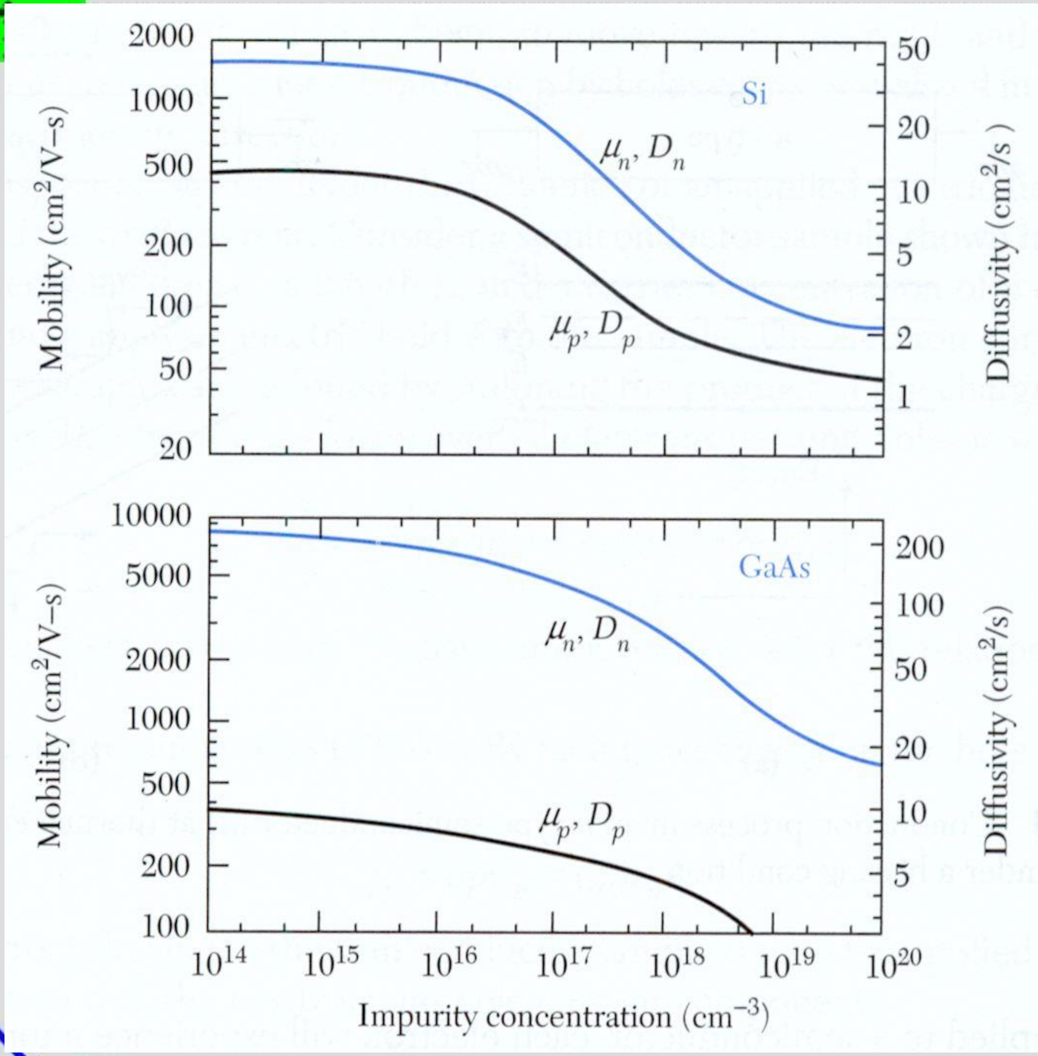
$$= n \cdot (\mu E) \cdot q$$

$$= n \cdot \mu \cdot q \cdot E$$

$$= \sigma E$$

$\Rightarrow \sigma$ (電導率，*Conductivity*) $= n \cdot \mu \cdot q$

$= \frac{1}{\rho}$ (電阻率，*Resistivity*)



摻雜濃度越高，移動率越低。

電子移動率

Si	**80~1700**
GaAs	**600~8400**

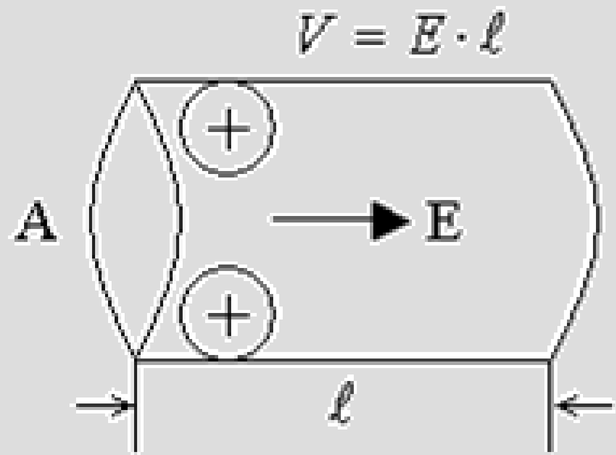
電洞移動率

Si	**40~450**
GaAs	**20~390**

☆ 巨觀公式之推導

$$V = E \cdot \ell \quad 【V = E \cdot d】$$

$$= \frac{1}{\sigma} \cdot \frac{I}{A} \cdot \ell \qquad \left[E = \frac{J}{\sigma} = \frac{\frac{I}{A}}{n \cdot \mu \cdot q} \right]$$



$$= \frac{1}{\sigma} \cdot \frac{\ell}{A} \cdot I$$

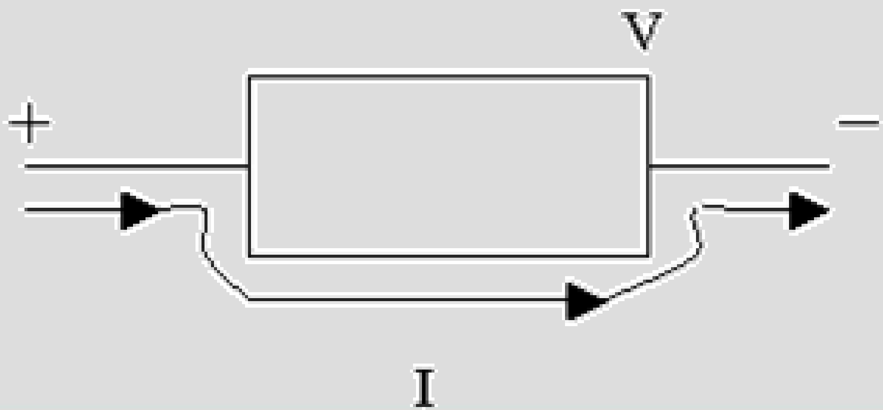
$$= RI \quad \Rightarrow \quad R = \rho \frac{\ell}{A}$$

歐姆定律 → 跨在電阻元件上電壓和流經此元件之電流成正比

$$\Rightarrow V \propto I$$

$$V = RI$$

$$R = \frac{V}{I}$$ 【Ω，歐姆(*ohm*)】



ρ(電阻率)決定材料的導電性，而非 R (電阻值)來決定

材料A

$$R_A = 2R_B$$

$$\Rightarrow \quad \frac{\ell_A}{A_A} = 1 \rightarrow \rho_A$$

材料B

$$R_B$$

$$\frac{\ell_B}{A_B} = 0.1 \rightarrow \rho_B$$

$$\rho_B = 5\rho_A$$

→ **材料B的** **電阻率高**
導電性差

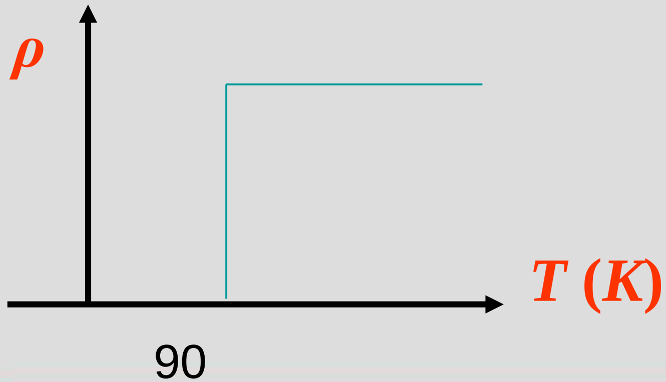
超導體(Superconductor) 0

導體(Conductor) $10^{-6} \sim 10^{-8}$

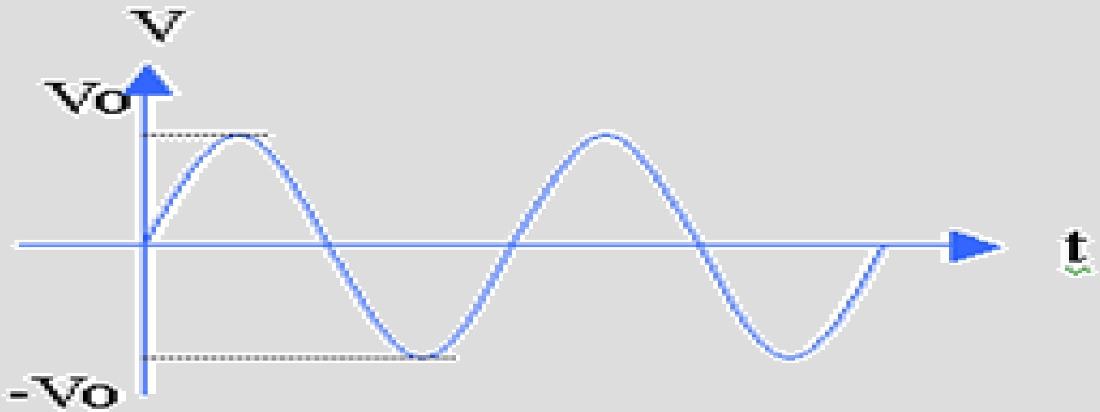
半導體(Semiconductor) 1~1000

絕緣體(Insulator) $> 10^{8}$

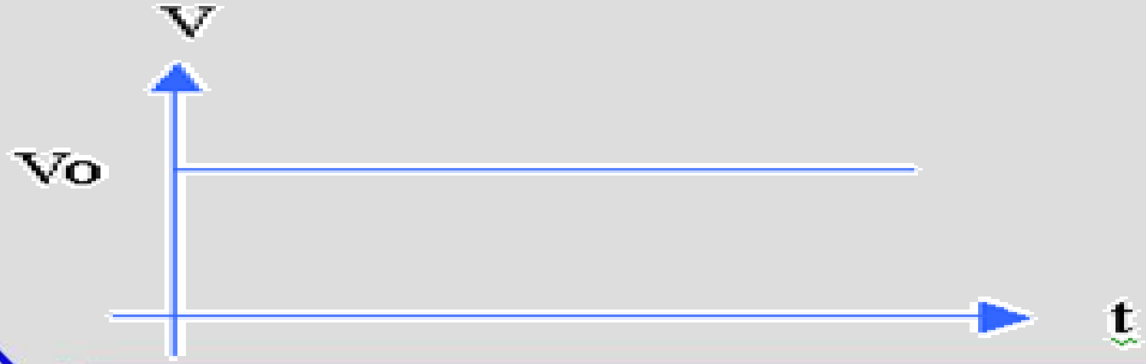
電阻率 $\rho(\Omega\text{-}m)$



交流(AC) → Alternating Current



直流(DC) → Direct Current



電阻消耗 － 電功率

$$P = \frac{\Delta W}{\Delta t} = \frac{-\Delta U}{\Delta t} = \frac{-(U_b - U_a)}{\Delta t}$$

$$= \frac{U_a - U_b}{\Delta t} = \frac{\Delta Q V_a - \Delta Q V_b}{\Delta t}$$

$$= \frac{\Delta Q}{\Delta t}(V_a - V_b)$$

$$= I \cdot V_{ab}$$

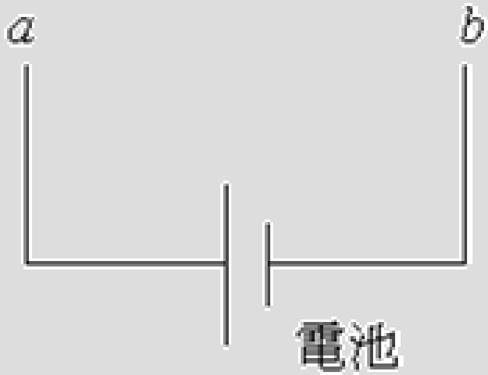
$$\Rightarrow P = I \cdot V$$

$$= I \cdot (IR) = I^2 R$$

$$= \left(\frac{V}{R}\right) V = \frac{V^2}{R}$$

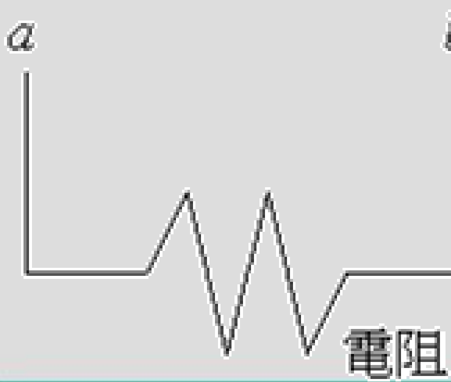
電動勢 (Electromotive Force)

→ 電池或電源所提供的電位 ξ



V_{ab} =電位差=電壓差=電動勢

$$P = I \cdot V_{ab} = I \cdot \xi$$



V_{ab} =電位差=電壓差~~=~~電動勢

消耗而不提供

交流電壓 $V = V_0 \sin\omega t$

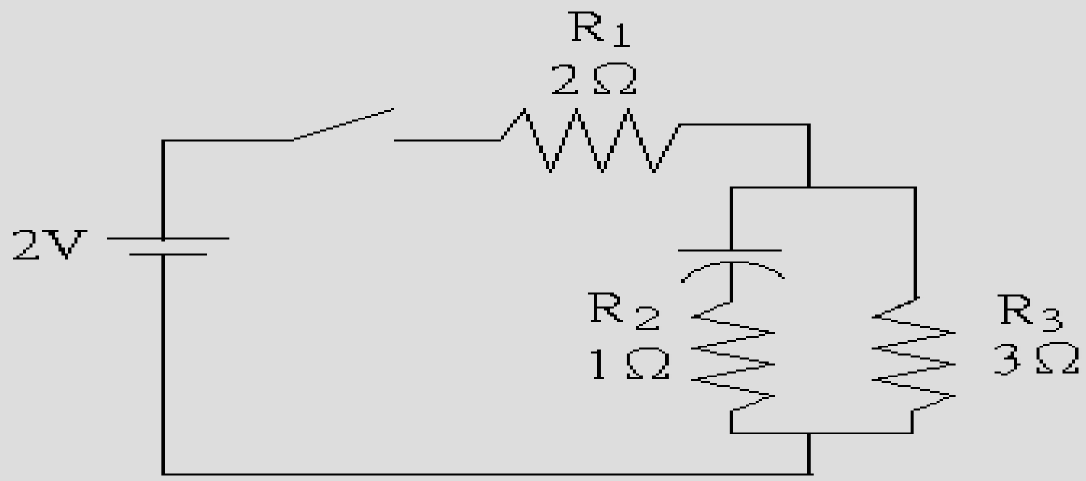
交流電流 $I = I_0 \sin\omega t$

電功率是以平均值去決定用電的瓦數或度數

$$\overline{P} = \overline{IV} = \overline{I^2}R \quad \Rightarrow \quad \overline{I^2} = \overline{I_0^{\,2}\sin^2\omega t} = I_0^{\,2}\,\overline{\sin^2\omega t} = \frac{1}{2}I_0^{\,2}$$

$$= \frac{\overline{V^2}}{R} \quad \Rightarrow \quad \overline{V^2} = \overline{V_0^{\,2}\sin^2\omega t} = \frac{1}{2}V_0^{\,2}$$

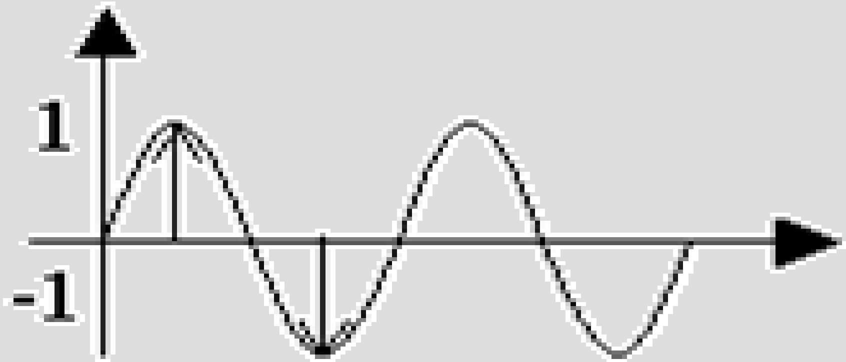
例15. 求瞬間通電時R_1、R_2、R_3電流？已充電完時R_1、R_2、R_3電流？電容的電壓？(C = 10pF) 儲存電荷？



請回家練習！

$$\overline{\sin\omega t} = \frac{最大+最小}{2}$$

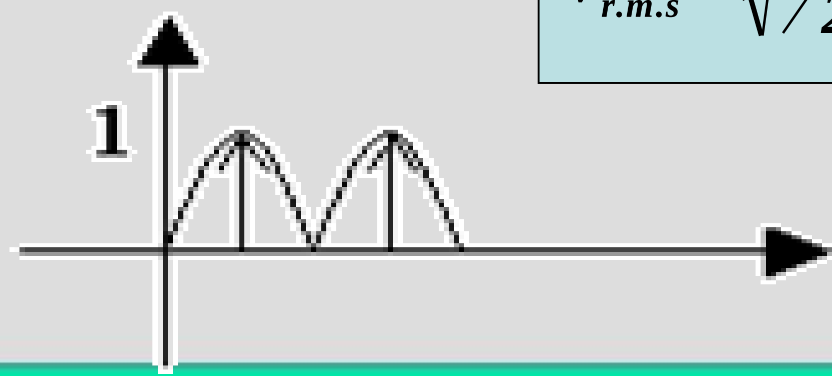
$$= \frac{1+(-1)}{2} = 0$$



$$I_{r.m.s} = \sqrt{\overline{I^2}} = \sqrt{1/2}\, I_0$$

$$V_{r.m.s} = \sqrt{1/2}\, V_0$$

$$\overline{\sin^2\omega t} = \frac{1+0}{2} = \frac{1}{2}$$



家庭用交流電

$$110V = V_{r.m.s}$$

$$V_0 = \sqrt{2}V_{r.m.s}$$

$$\overline{P} = \overline{I^2}R = I_{r.m.s}^2 \cdot R$$

$$= \frac{\overline{V^2}}{R} = \frac{V_{r.m.s}^2}{R}$$

$$I_{r.m.s} = \sqrt{\overline{I^2}} \Rightarrow I_{r.m.s}^2 = \overline{I^2}$$

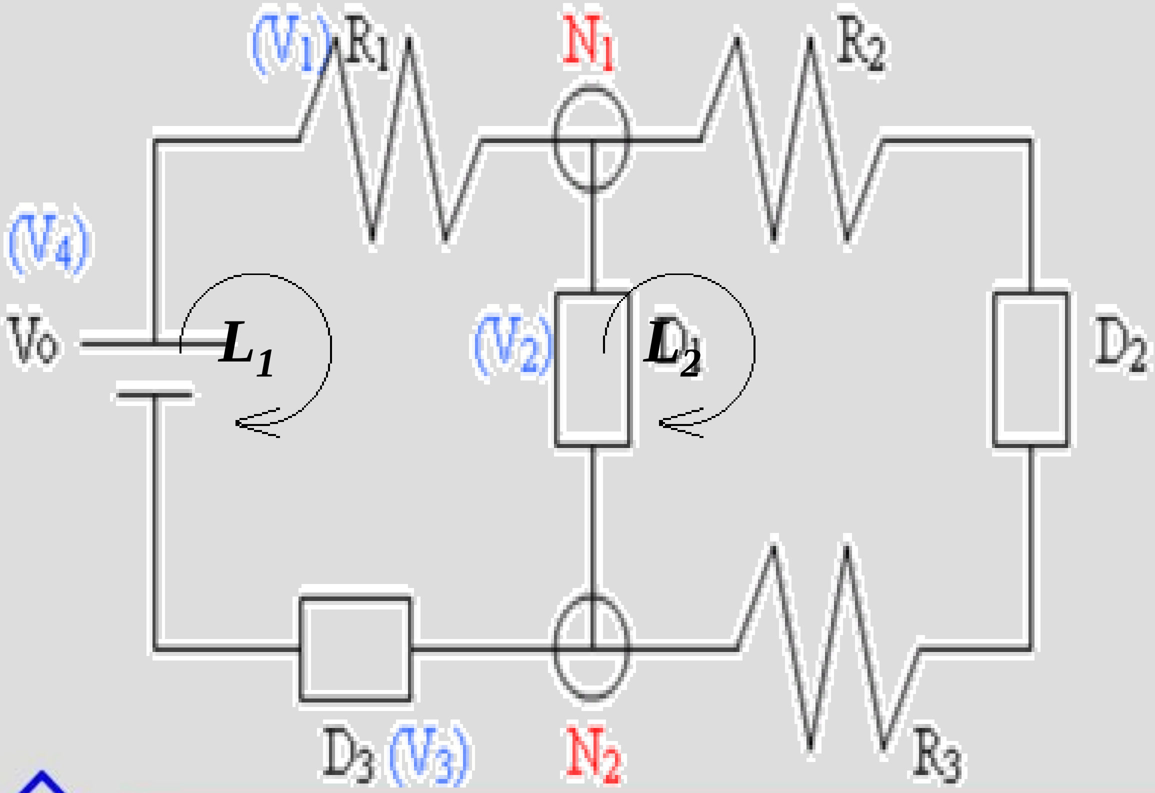
$$V_{r.m.s} = \sqrt{\overline{V^2}} \Rightarrow V_{r.m.s}^2 = \overline{V^2}$$

例16：家電110V，消耗1仟瓦，求電流峰值？

請回家練習！

3. 直流電路(DC Circuit)

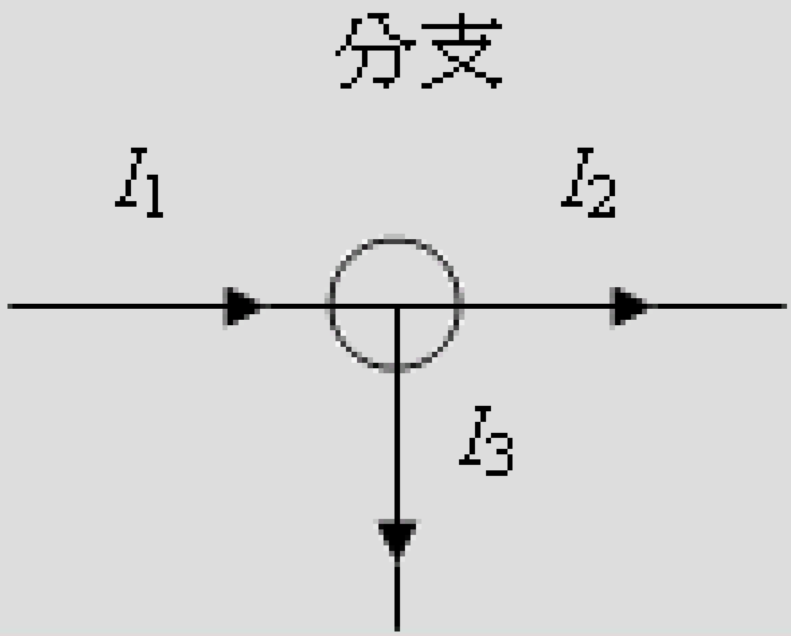
→ (1) 迴路(Loop) → 電路上各元件串接成一封閉路徑



$L_1 \rightarrow V_0$，R_1，D_1，D_3
$L_2 \rightarrow D_1$，R_2，D_2，R_3
$L_3 \rightarrow V_0$，R_1，R_2，D_3，
R_3，D_3

(2) 節點 (*Branch Point or Node*)

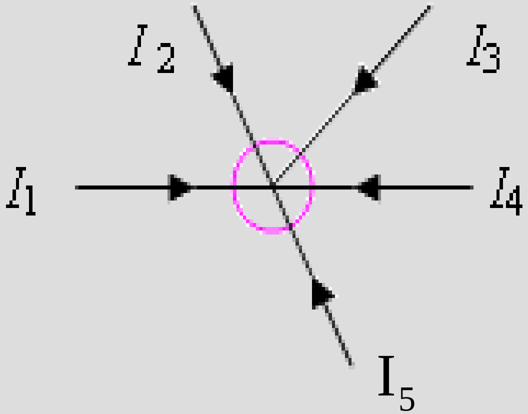
→ 電路上電流的分支點



☆ Kirchhoff's Rules

1st Rule KCL

流進某節點之電流總和為零



$$\sum_{i=1}^{5} I_i = 0$$

$$\Rightarrow I_1 + I_2 + I_3 + I_4 = -I_5$$

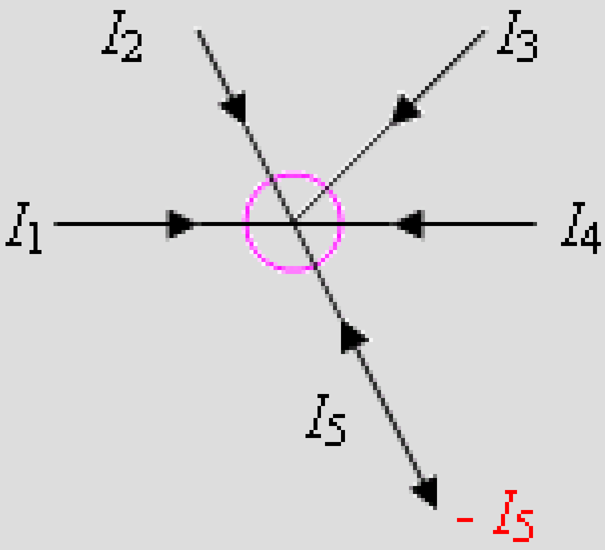
$$(\text{流進}) = (\text{流出})$$

$$\Rightarrow I_1 + I_2 + I_3 + I_4 + I_5 = 0$$

定義：

電流流進節點 → 電流為正

電流流出節點 → 電流為負



2nd Rule KVL

沿著迴路電流方向各元件上之電壓總和為零

$$\sum_{i}^{4} V_i = 0$$

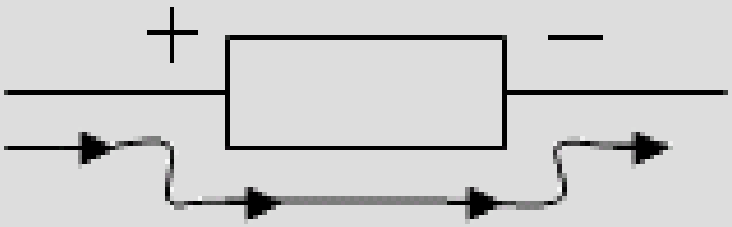
$$\Rightarrow \quad V_0 = V_1 + V_2 + V_3 = -V_4$$

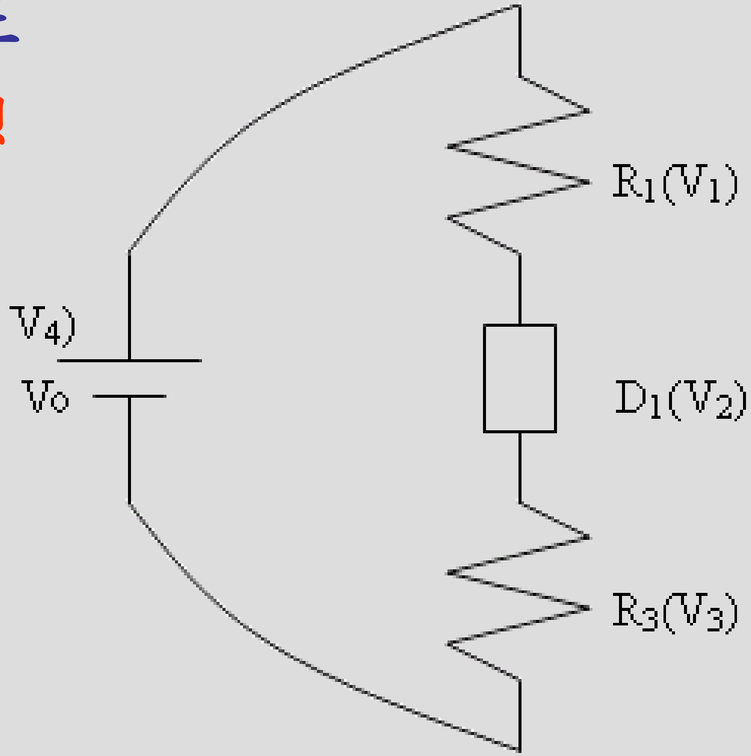
$$\Rightarrow \quad V_1 + V_2 + V_3 + V_4 = 0$$

定義 ：

電流流進元件 → 電壓為正

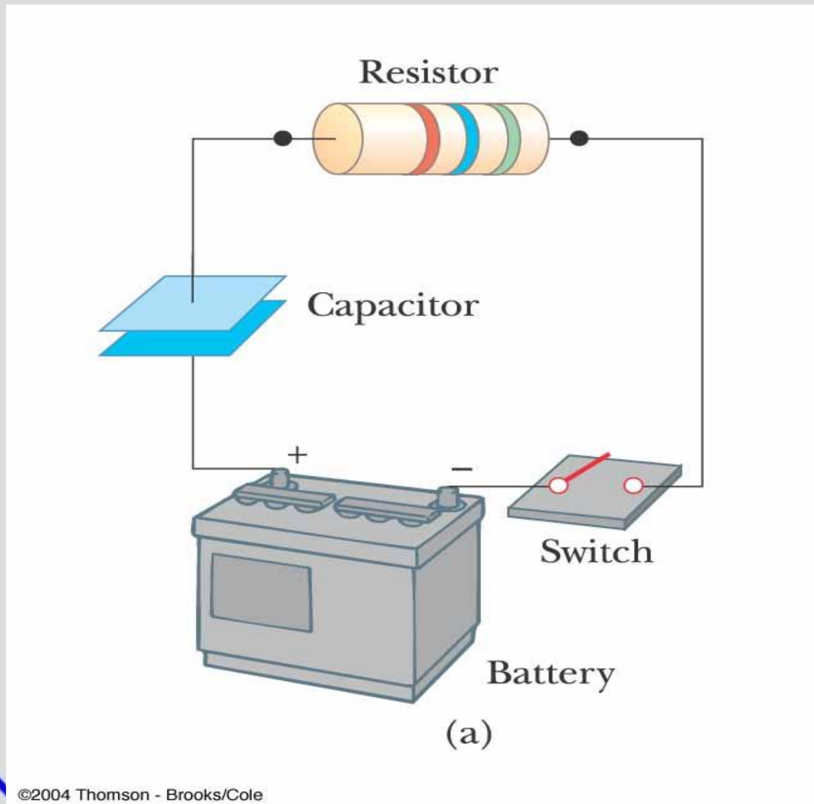
電流流出元件 → 電壓為負

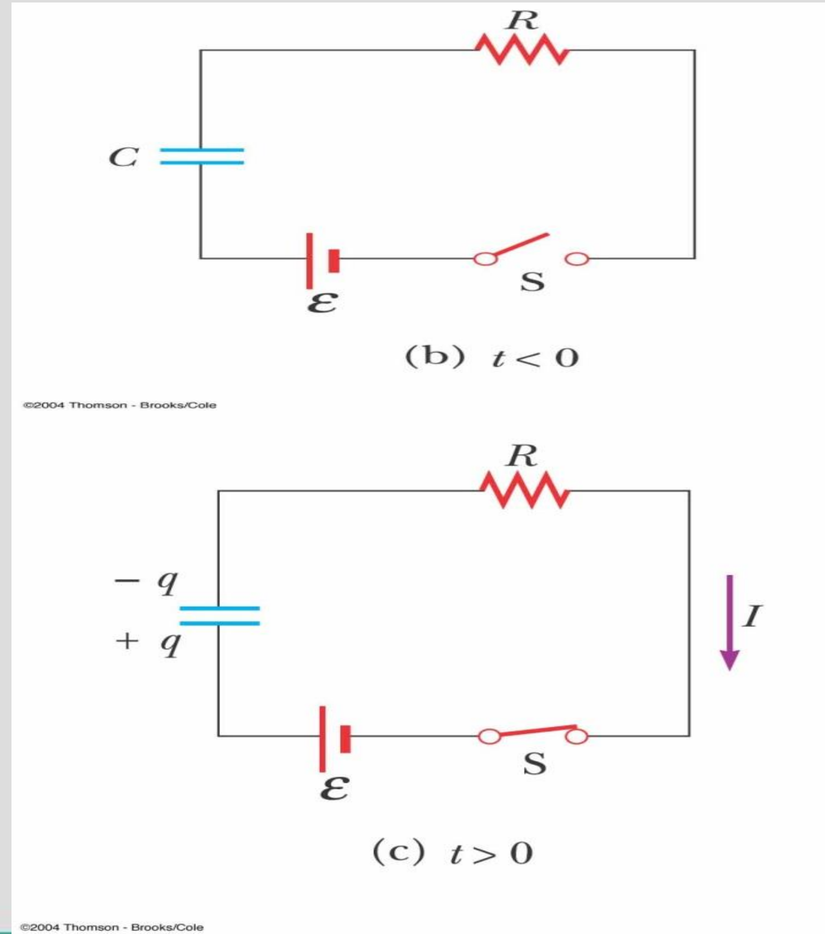




RC Circuits

- A circuit containing a series combination of a resistor and a capacitor





Charging a Capacitor

$$\varepsilon - \frac{q}{C} - IR = 0$$

Initial current: $$I_o = \frac{\varepsilon}{R}$$

Max. charge $$Q = C\varepsilon$$

Charge and Current vs. Time

$$I = \frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC}$$

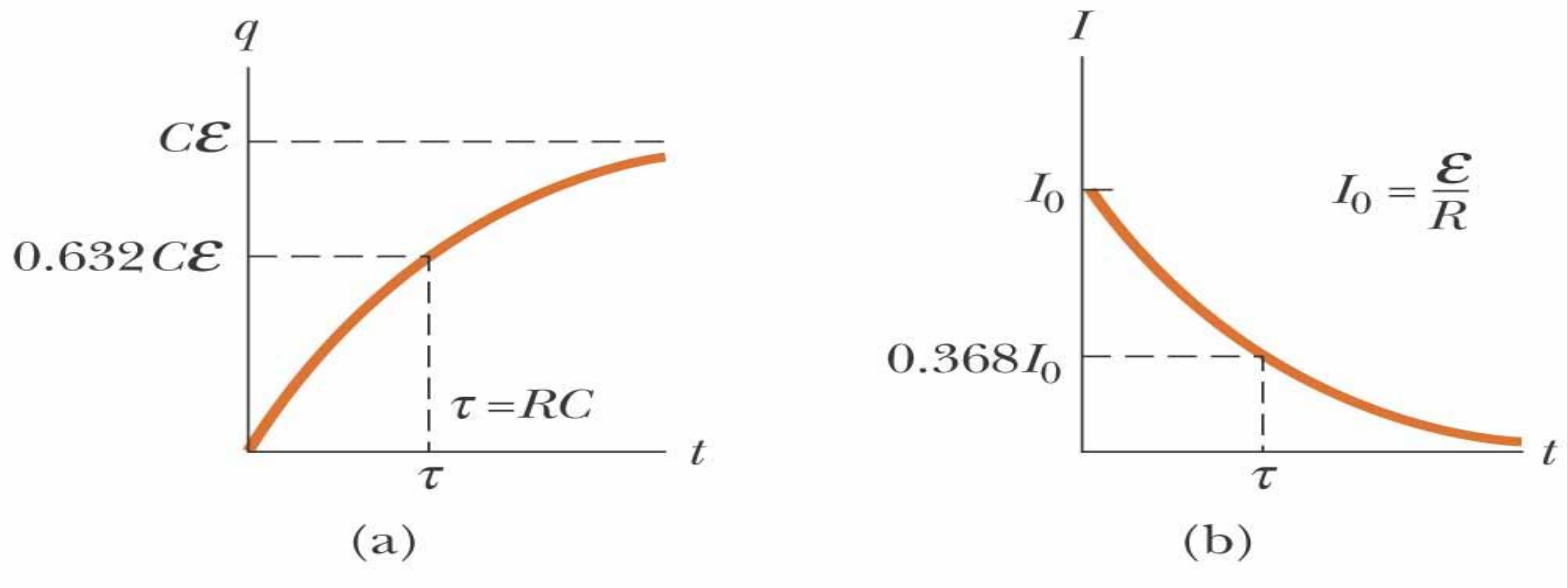
$$\int_0^q \frac{dq}{(q - C\varepsilon)} = -\frac{1}{RC}\int_0^t dt$$

$$q(t) = C\varepsilon(1 - e^{-t/\tau}) = Q(1 - e^{-t/\tau})$$

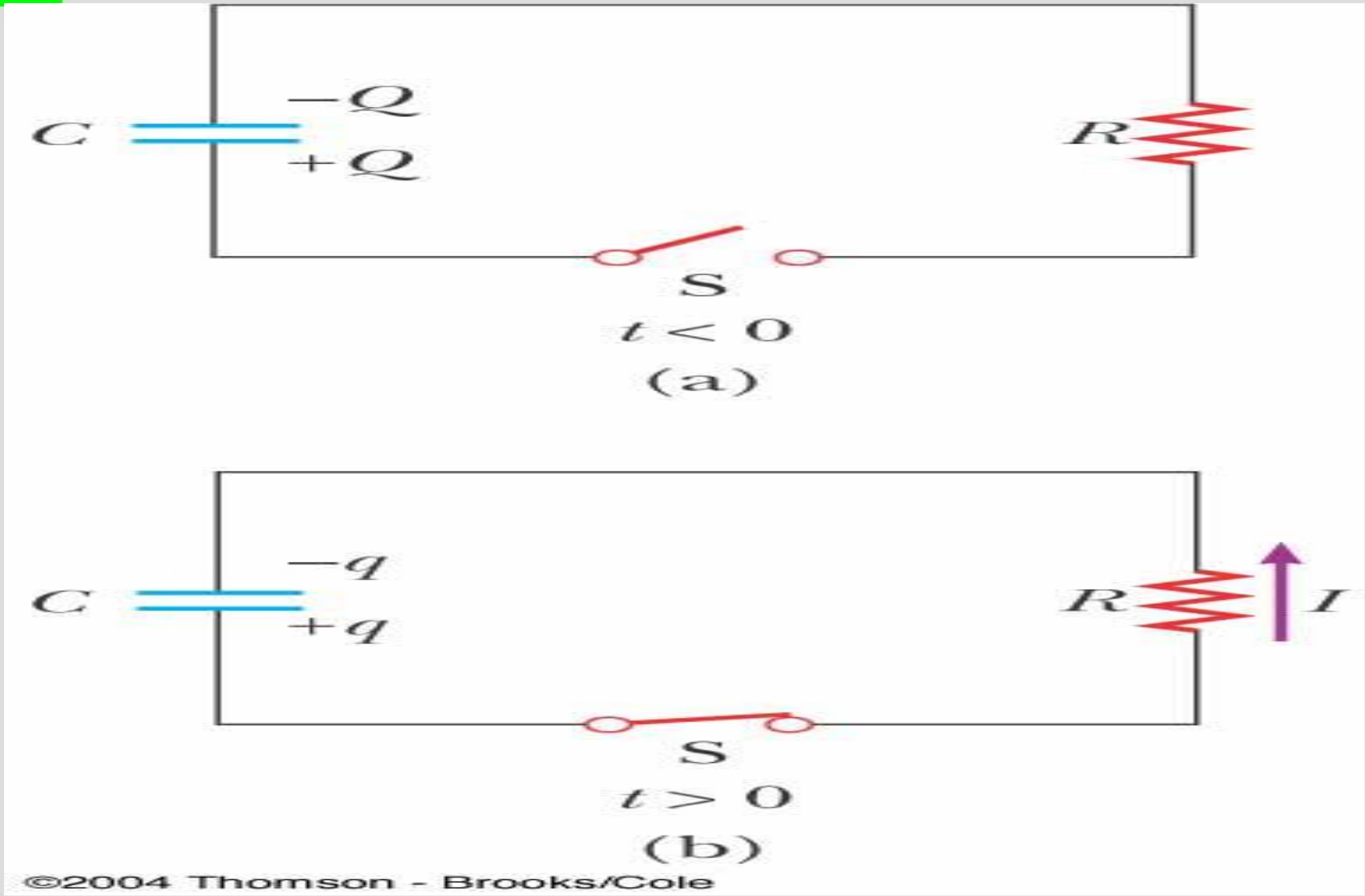
$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

where τ = time constant = RC



Discharging a Capacitor



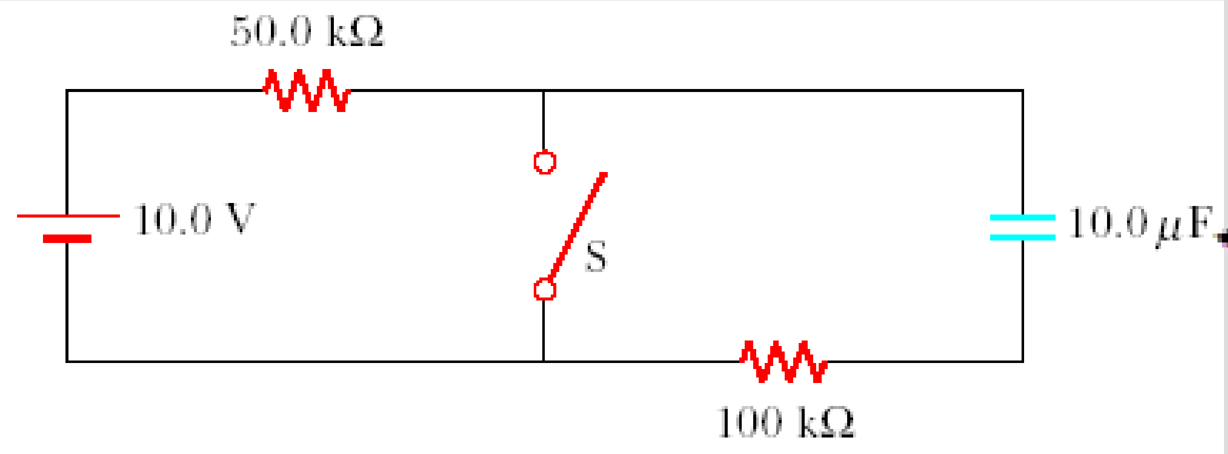
Charge and Current vs. Time

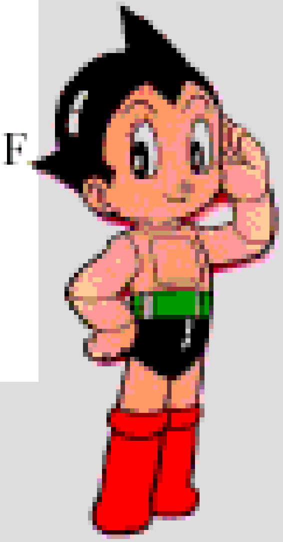
$$-\frac{q}{C} - IR = -\frac{q}{C} - \frac{dq}{dt}R = 0 \qquad \frac{dq}{q} = -\frac{1}{RC}dt$$

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt \qquad \ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$q(t) = Qe^{-t/\tau} \qquad I(t) = -\frac{Q}{RC}e^{-t/\tau}$$

- **例17.** In the RC circuit, the switch S has been open for a long time. It is then suddenly closed. Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) Let the switch be closed at $t = 0$. Determine the current in the switch as a function of time.





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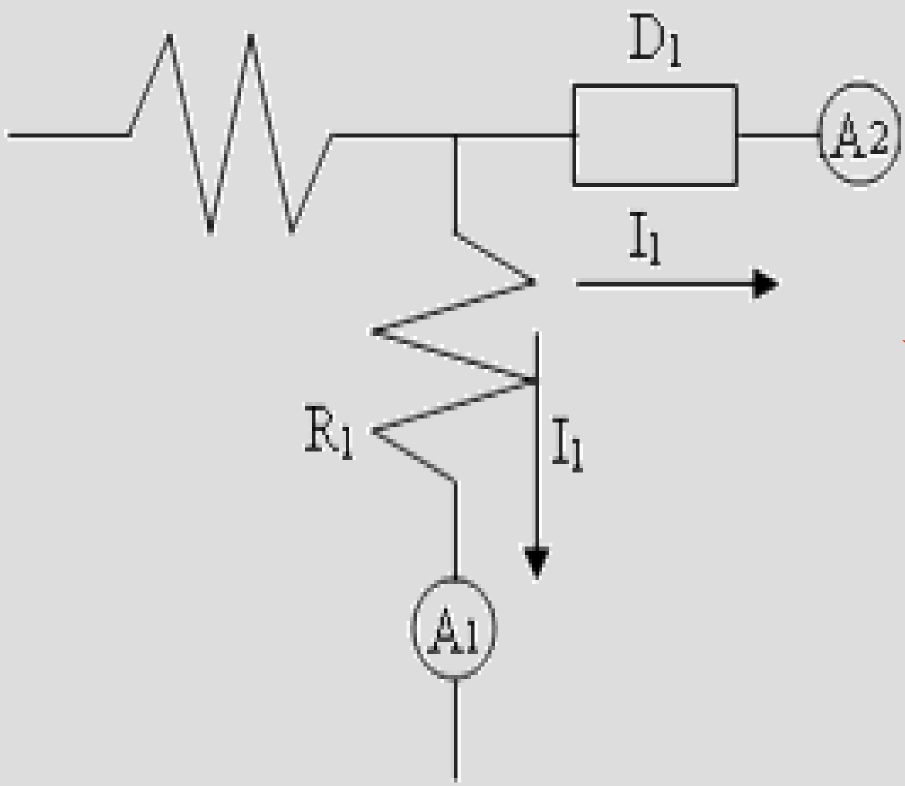
☆ 安培計 (Ammeter) → 測量電流



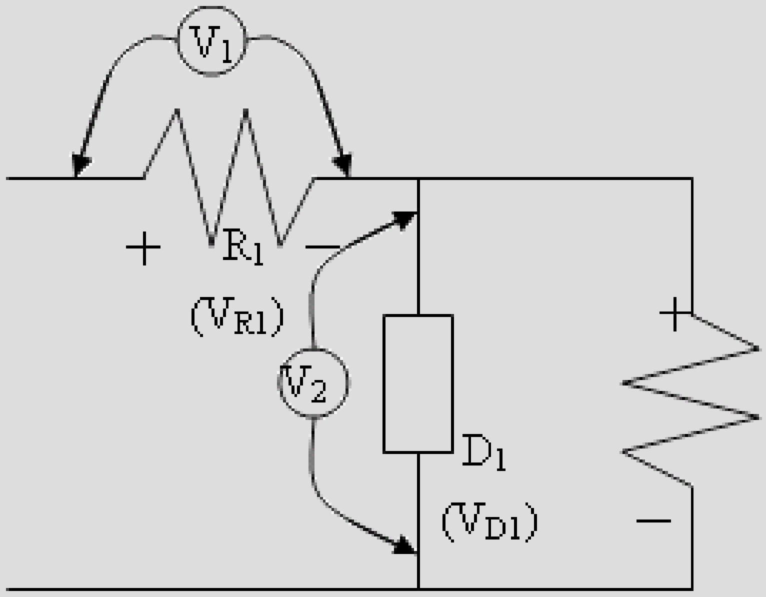
測量流經R_1之電流　　$I_1 \rightarrow A_1$

測量流經D_1之電流　　$I_2 \rightarrow A_2$

☆與待測元件**串聯**

I_2

☆伏特計(Voltmeter) → 測量電壓

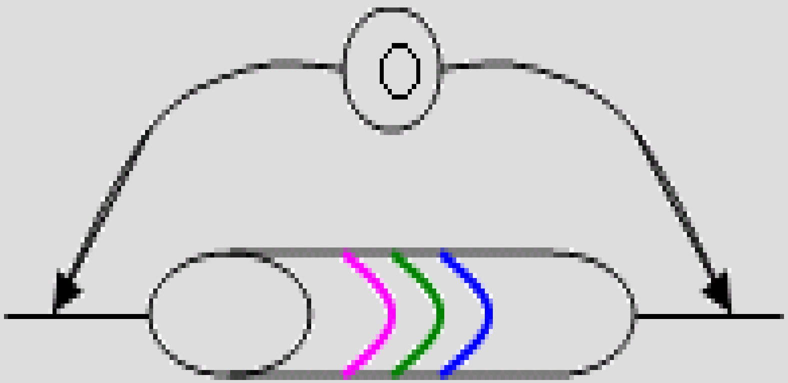


測量流經R_1之電壓　　$V_{R1} \rightarrow V_1$

測量流經D_1之電壓　　$V_{D1} \rightarrow V_2$

☆與待測元件**並聯**

☆歐姆計(Ohmmeter) → 測量電阻



☆ 與待測元件並聯